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Academy of Scientific Research & Technology

Joint Institute for Nuclear Research
SCIENCE BRINGS NATIONS TOGETHER

Laboratory of Information Technologies (LIT) projects

Numerical and analytical calculations in gravitation and cosmology

Analyzing Properties of Compact _ Charged Astronomical object Using Theory of General Relativity

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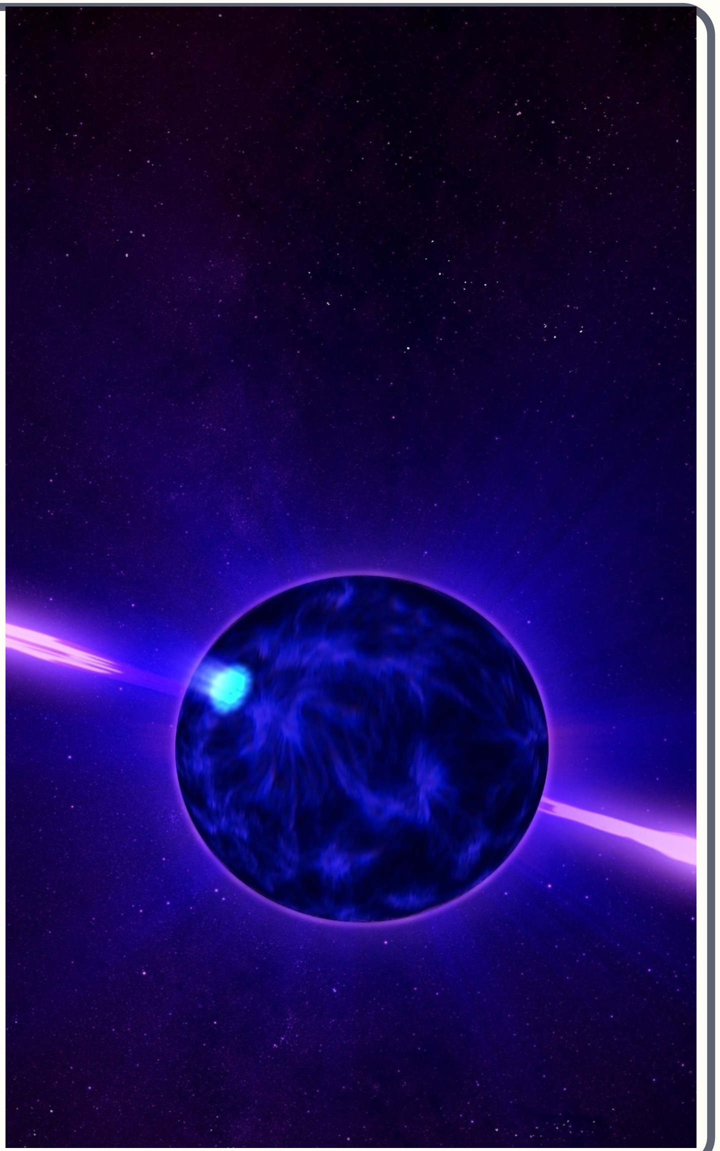
Numerical and analytical calculations in gravitation and cosmology

Training Task:

Analyzing Properties of Compact _ Charged Astronomical object Using Theory of General Relativity



Analyzing Properties of Compact _ Charged Astronomical object Using Theory of General Relativity

We can list our training tasks as:

Introduce a case study in physics, specially in cosmology or gravity.

1. *What is the interest in this case? (Compact objects like BH open new area of discussion where many physical concepts discussed)*
2. *What we get from its discussion? (We can describe the prop of BH and give a model how it may communicate with its surrounding)*
3. *What scientific background do you need to make this study? (GR and Differential geometry)*

Analyzing Properties of Compact _ Charged Astronomical object Using Theory of General Relativity

We can list our training tasks as:

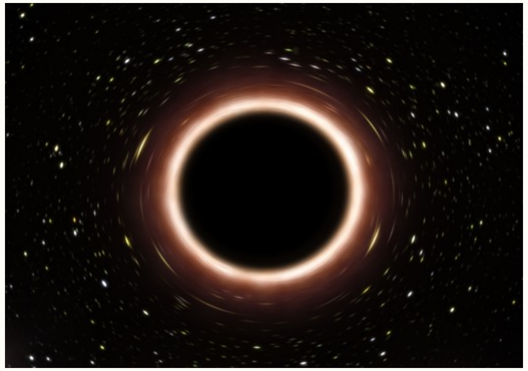
How we can discuss this case.

1. *Collect all possible models or data from previse work.*
2. *Define the initial conditions, those may used in our model.*
3. *Put/Find theoretical model that may used in the model.*
4. *Build an automated code (Mathematica code) that aim to reduce the human error in calculations, push the process to work fast, link all model items with each other; as if you change the initial conditions all results and calculations change*
5. *Compare the new model with previse studies and represent results in graphs to see they behavior.*

Compact Charged Astronomical Object

Properties of this Object

- It's charged astronomical objects.
- This type has a quit large mass > 1.4 $M_{\odot}$.
- We suppose that, it's a static object.
- Examples of this type of object are White dwarf and Black holes.

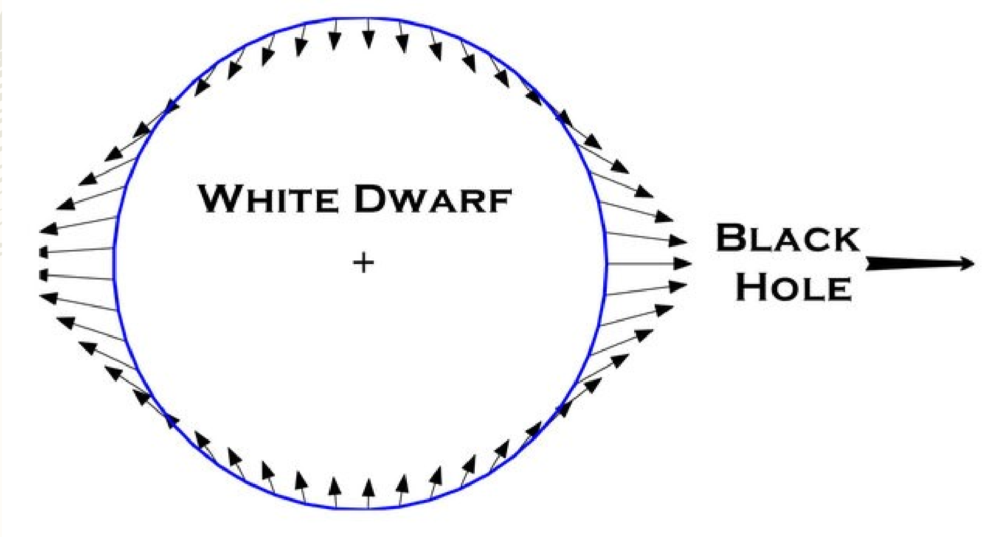


Compact Charged Astronomical Object

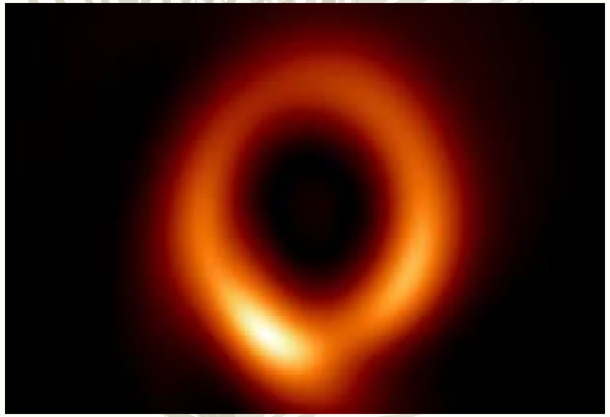
Why this type?

One of the most accepted ideas in Astrophysics discuss the creation of black hole say that:

"The most massive stars, with $\geq 8\, M_{\odot}$, will never become white dwarfs. Instead, at the end of their lives, white dwarfs will explode in a violent supernova, leaving behind a neutron star or black hole"

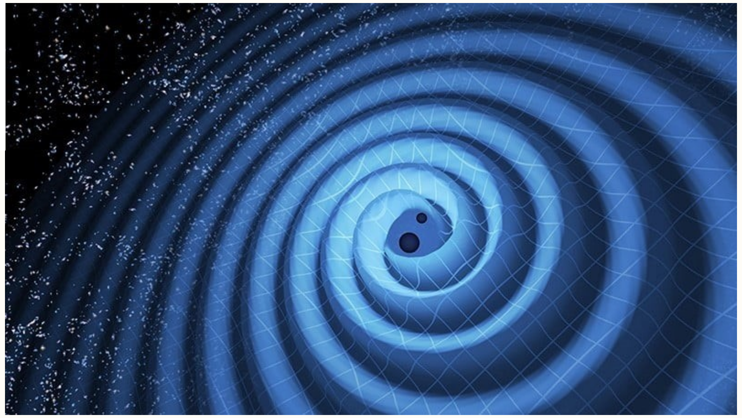


So this study may introduce the creation of black hole as a evolution in Compact charged astronomical object, and so discuss the properties of this mystery object.



EHT take the 1st photo of Black hole (2022)

Note that black hole became reality after LIGO project results and EHT observations



LIGO detect gravitational wave of two black hole emerging (2019)

Compact Charged Astronomical Object

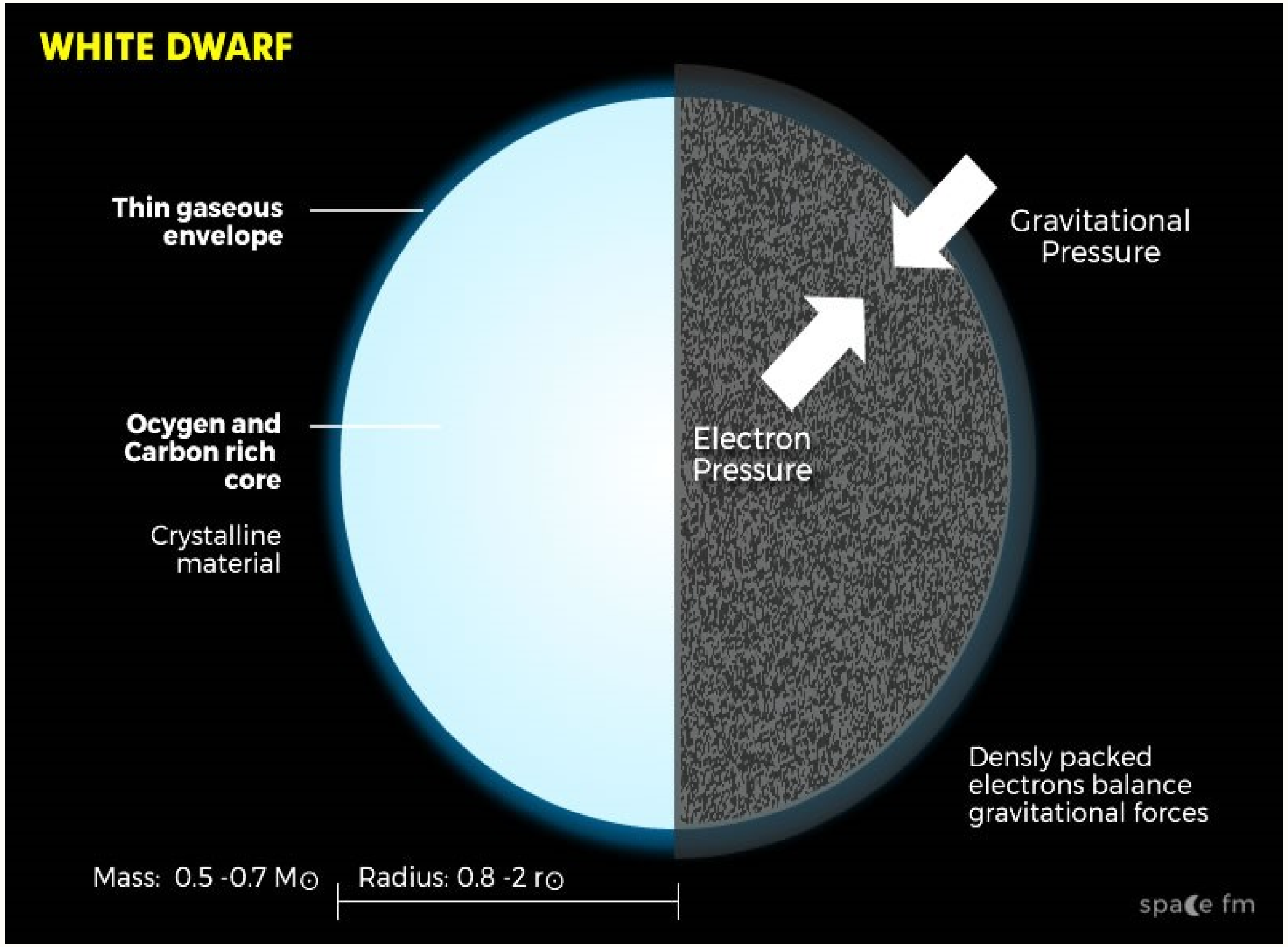
The potential field equations:

This object describe by:

- Electromagnetic field that describes the electric charge properties.

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ as $A = \{0,0,0,\rho\}$

- Scalar field that describe the mass of this object.



General Theory of Relativity

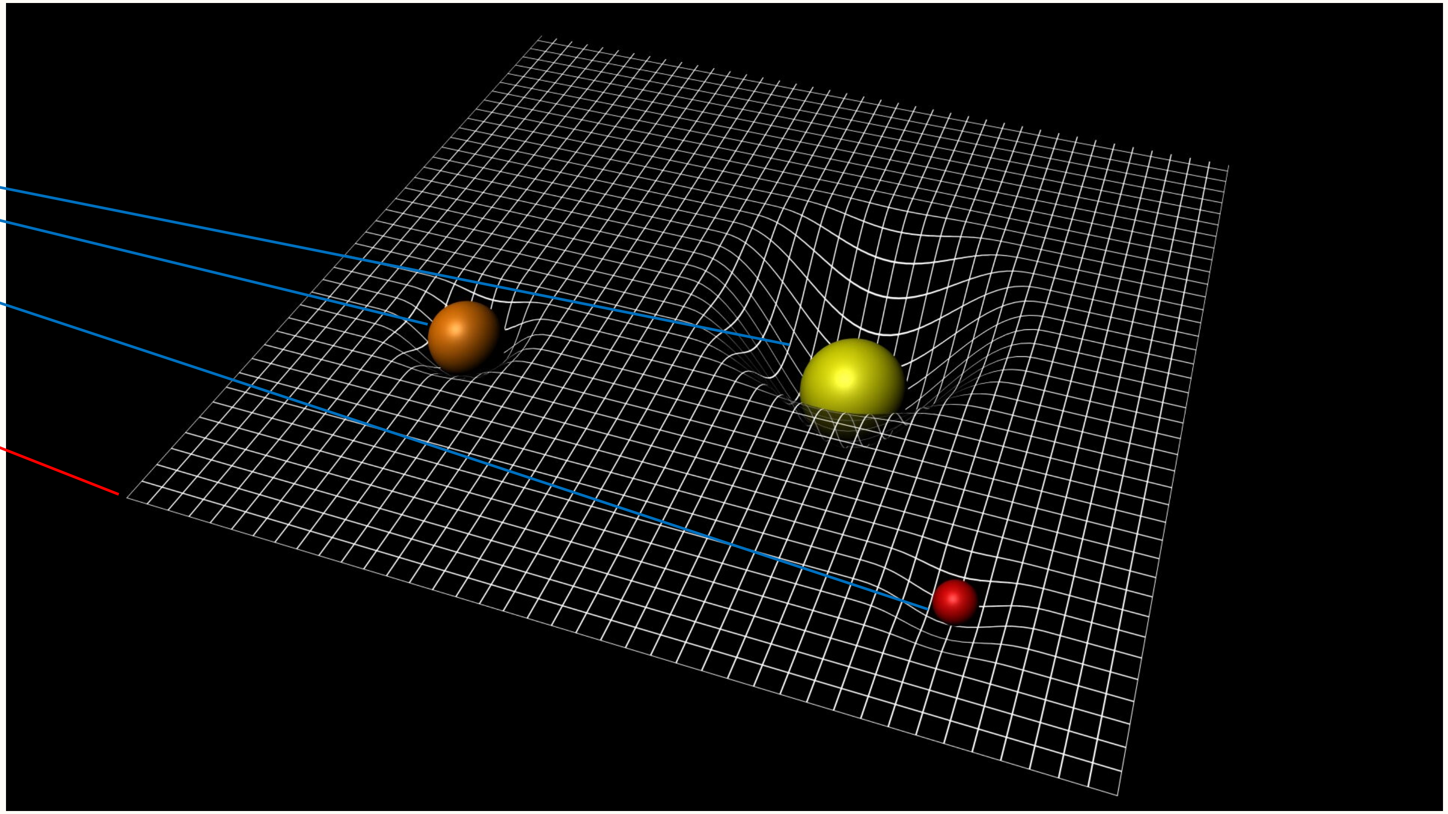
The Einstein field equations:

In the general theory of relativity, the Einstein field equations relate the geometry of spacetime to the distribution of matter within it.

$$G_{\mu\nu} = \kappa T_{\mu\nu}$$

The spacetime show the matter how to move.

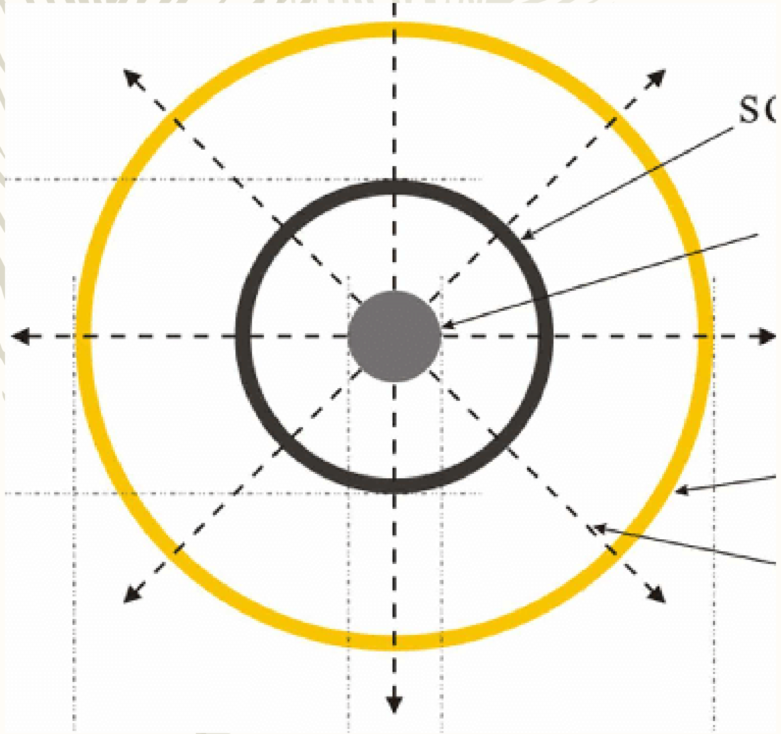
The matter show the spacetime how to deform.

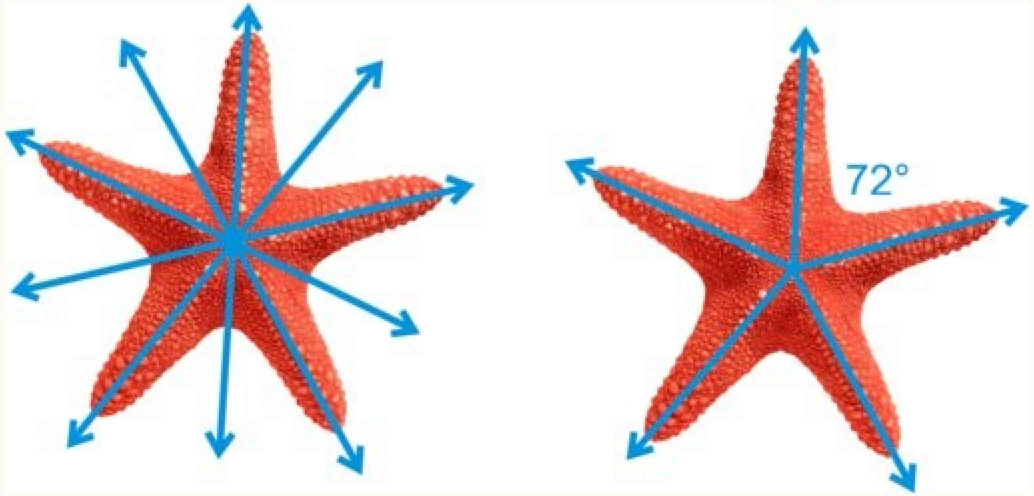


General Theory of Relativity

The spherical symmetric metric:

$$ds^2 = e^{2\gamma(r)}dt^2 - e^{2\alpha(r)}dr^2 - e^{2\beta(r)}(d\theta^2 + \sin\theta^2\, d\phi^2)$$

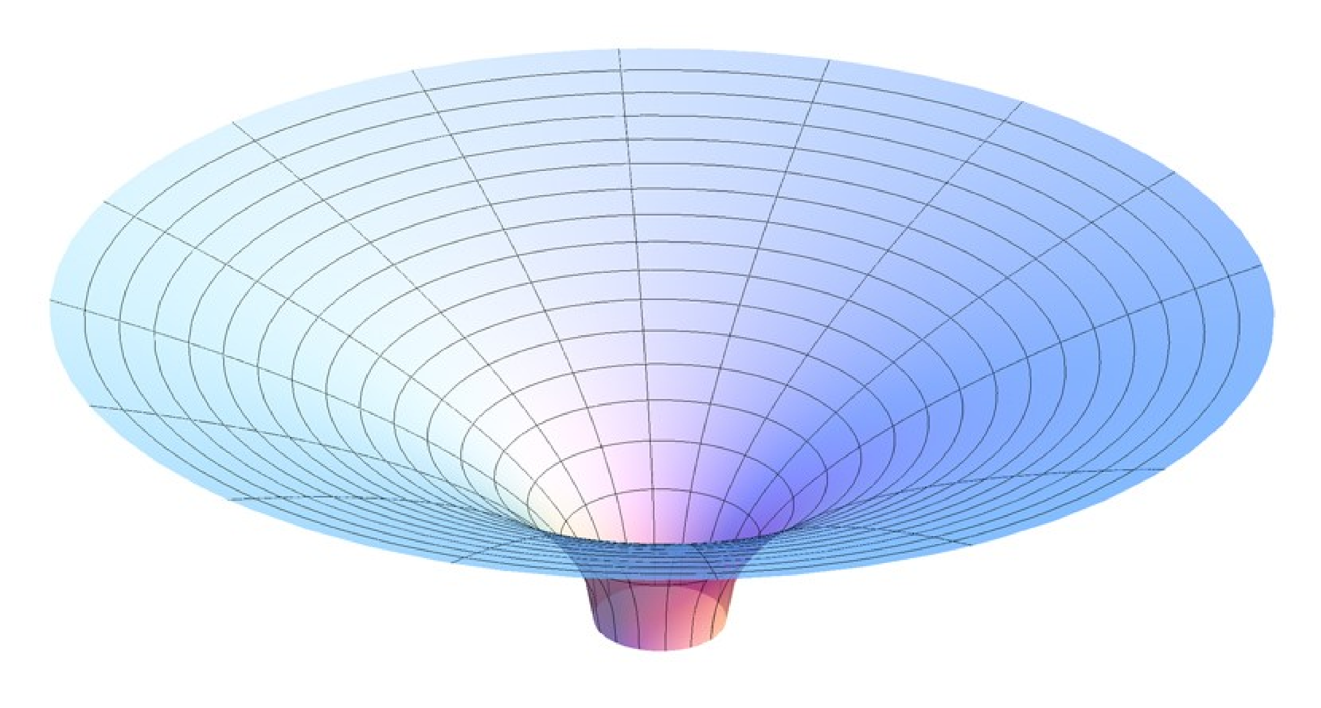




General Theory of Relativity

The Schwarzschild metric:

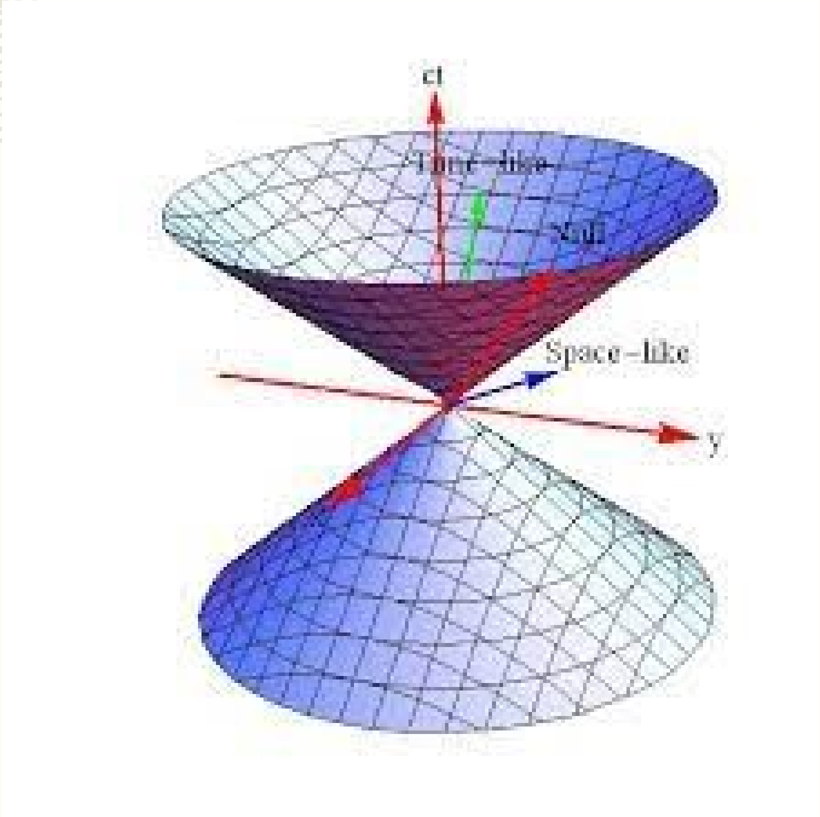
$$ds^2 = C^2\left(1 - \frac{R_s}{r}\right)dt^2 - \frac{dr^2}{\left(1 - \frac{R_s}{r}\right)} - r^2(d\theta^2 + \sin\theta^2\, d\phi^2)$$



General Theory of Relativity

The Minkowski metric:

$$ds^2 = C^2dt^2 - dr^2 - r^2(d\theta^2 + \sin\theta^2 \, d\phi^2)$$



The steps of our work

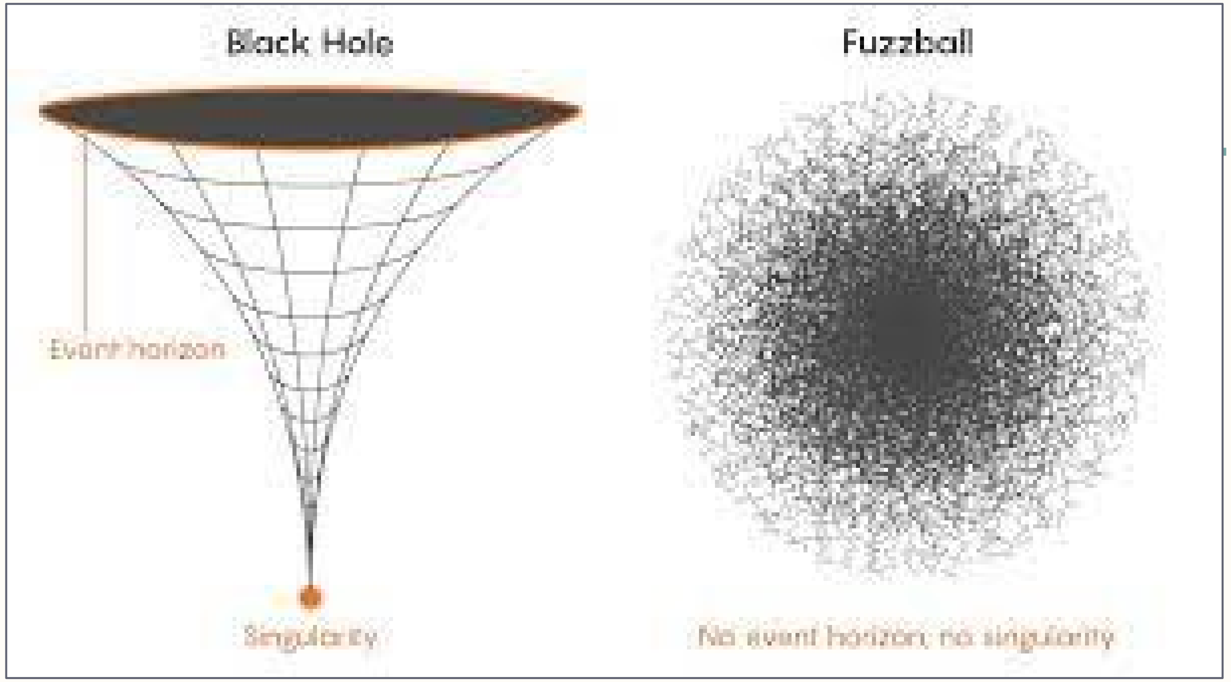
- Define the field equations describe potential of this compact object.
- Introduce the spherical symmetrical metric that will describe the Spacetime around this object
- We will take two different cases in order to discuss this metric.
- We will define ($\gamma = -\alpha$) to discuss isometric coordinates, and ($\gamma = \alpha + 2\beta$) for harmonic coordinates
- Drive the LHS of Einstein field equation (Einstein metric) using the above metric.
- Drive the RHS of Einstein field equations (Stress energy tensor) using the field equations.
- Introduce a Mathematica code that can drive the solutions of Einstein field equations, and present its solution graphically.
- Discuss the behavior of the spherical symmetrical metric with Schwarzschild and Minkowski metrics

The isometric coordinates

The General equations:

$$\gamma = -\alpha$$

The term "isometric" comes from the Greek for "equal measure", reflecting that the scale along each axis of the projection is the same.

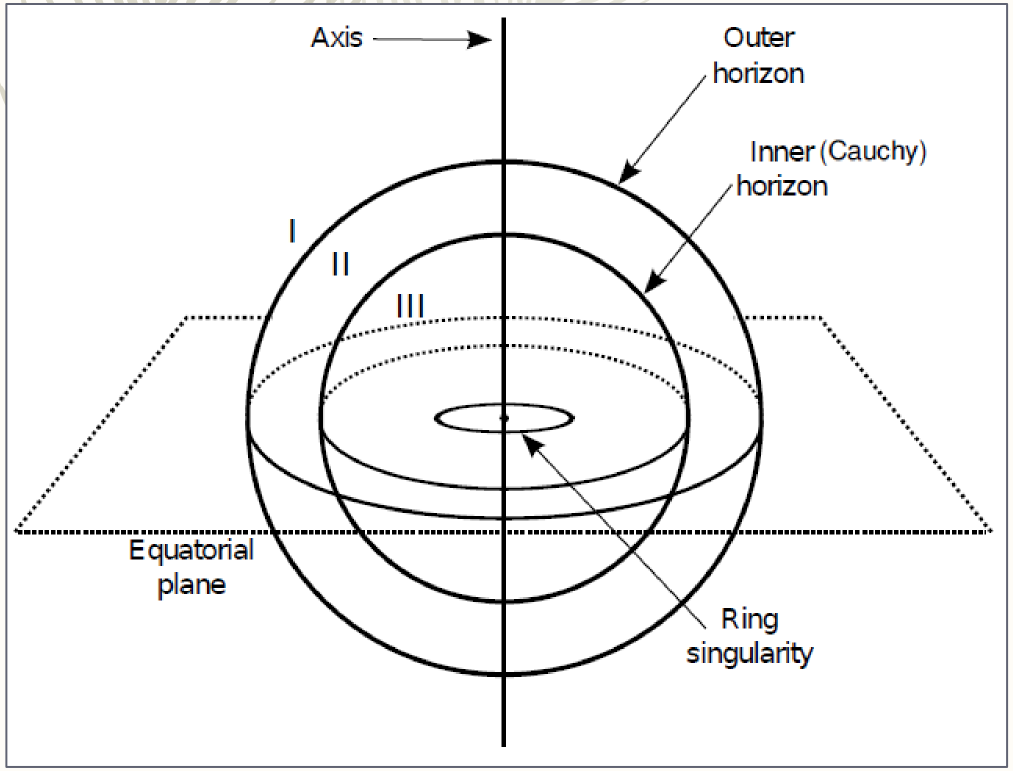


This type of coordinates is very useful in describing, the very massive objects.

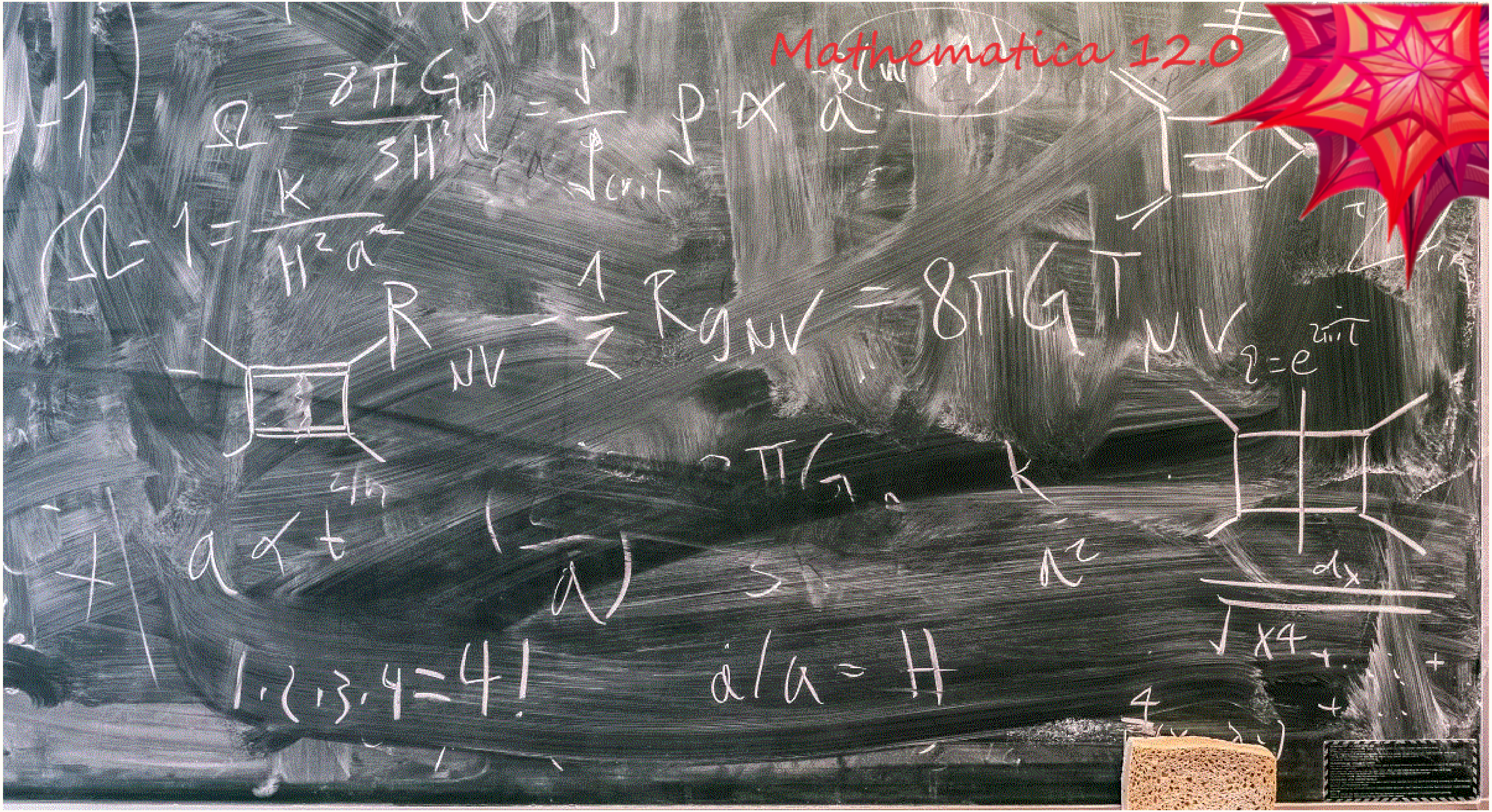
General Theory of Relativity

The spherical symmetric metric in isometric coordinates:

$$ds^2 = e^{2\gamma(r)}dt^2 - e^{-2\gamma(r)}dr^2 - e^{2\beta(r)}(d\theta^2 + \sin\theta^2\, d\phi^2)$$



$e^{2\gamma[r]}$	0	0	0
0	$-e^{-2\gamma[r]}$	0	0
0	0	$-e^{2\beta[r]}$	0
0	0	0	$-e^{2\beta[r]}\,\mathrm{Sin}[\theta]^2$


Mathematica 12.0
1·2·3·4=4!
ȧ/a = H

The Christoffel symbols from the Code

The General equations:

$$\Gamma^{\lambda}{}_{\mu\nu} = \frac{1}{2}\mathcal{G}^{\lambda i}\left(\frac{\partial \mathcal{G}_{i\mu}}{\partial \chi^{\nu}} + \frac{\partial \mathcal{G}_{i\nu}}{\partial \chi^{\mu}} - \frac{\partial \mathcal{G}_{\mu\nu}}{\partial \chi^{i}}\right)$$

The Output of Mathematica Code

$\Gamma^{1}{}_{\{2,1\}}$	$\gamma'[r]$
$\Gamma^{2}{}_{\{1,1\}}$	$e^{4\gamma[r]}\gamma'[r]$
$\Gamma^{2}{}_{\{2,2\}}$	$-\gamma'[r]$
$\Gamma^{2}{}_{\{3,3\}}$	$-e^{2(\beta[r]+\gamma[r])}\beta'[r]$
$\Gamma^{2}{}_{\{4,4\}}$	$-e^{2(\beta[r]+\gamma[r])}\text{Sin}[\theta]^2\beta'[r]$
$\Gamma^{3}{}_{\{3,2\}}$	$\beta'[r]$
$\Gamma^{3}{}_{\{4,4\}}$	$-\text{Cos}[\theta]\,\text{Sin}[\theta]$
$\Gamma^{4}{}_{\{4,2\}}$	$\beta'[r]$
$\Gamma^{4}{}_{\{4,3\}}$	$\text{Cot}[\theta]$

The Riemann tensor from the Code

The General equations:

$$R^{\lambda}{}_{\omega\nu\mu} = \frac{\partial \Gamma^{\lambda}{}_{\mu\omega}}{\partial \chi^{\nu}} - \frac{\partial \Gamma^{\lambda}{}_{\mu\nu}}{\partial \chi^{\omega}} + \sum_{s} \left(\Gamma^{s}{}_{\mu\omega} \Gamma^{\lambda}{}_{\nu s} - \Gamma^{s}{}_{\mu\nu} \Gamma^{\lambda}{}_{\omega s} \right)$$

The Output of Mathematica Code

$R^1{}_{\{2,2,1\}}$	$2\gamma'[r]^2 + \gamma''[r]$
$R^1{}_{\{3,3,1\}}$	$e^{2(\beta[r]+\gamma[r])}\beta'[r]\gamma'[r]$
$R^1{}_{\{4,4,1\}}$	$e^{2(\beta[r]+\gamma[r])}\mathrm{Sin}[\theta]^2\beta'[r]\gamma'[r]$
$R^2{}_{\{1,2,1\}}$	$e^{4\gamma[r]}\left(2\gamma'[r]^2 + \gamma''[r]\right)$
$R^2{}_{\{3,3,2\}}$	$e^{2(\beta[r]+\gamma[r])}\left(\beta'[r]^2 + \beta'[r]\gamma'[r] + \beta''[r]\right)$
$R^2{}_{\{4,4,2\}}$	$e^{2(\beta[r]+\gamma[r])}\mathrm{Sin}[\theta]^2\left(\beta'[r]^2 + \beta'[r]\gamma'[r] + \beta''[r]\right)$
$R^3{}_{\{1,3,1\}}$	$e^{4\gamma[r]}\beta'[r]\gamma'[r]$
$R^3{}_{\{2,3,2\}}$	$-\beta'[r]^2 - \beta'[r]\gamma'[r] - \beta''[r]$
$R^3{}_{\{4,4,3\}}$	$\mathrm{Sin}[\theta]^2\left(-1 + e^{2(\beta[r]+\gamma[r])}\beta'[r]^2\right)$
$R^4{}_{\{1,4,1\}}$	$e^{4\gamma[r]}\beta'[r]\gamma'[r]$
$R^4{}_{\{2,4,2\}}$	$-\beta'[r]^2 - \beta'[r]\gamma'[r] - \beta''[r]$
$R^4{}_{\{3,4,3\}}$	$1 - e^{2(\beta[r]+\gamma[r])}\beta'[r]^2$

The Ricci tensor from the Code

The General equations:

$$R_{\omega\mu} = R^{\lambda}{}_{\omega\lambda\mu}$$

The Output of Mathematica Code

$$\mathcal{R}_{\{1,1\}} \quad e^{4\gamma[r]}\left(2\beta'[r]\gamma'[r] + 2\gamma'[r]^2 + \gamma''[r]\right)$$

$$\mathcal{R}_{\{2,2\}} \quad -2\beta'[r]^2 - 2\beta'[r]\gamma'[r] - 2\gamma'[r]^2 - 2\beta''[r] - \gamma''[r]$$

$$\mathcal{R}_{\{3,3\}} \quad 1 - 2e^{2(\beta[r]+\gamma[r])}\beta'[r]^2 - 2e^{2(\beta[r]+\gamma[r])}\beta'[r]\gamma'[r] - e^{2(\beta[r]+\gamma[r])}\beta''[r]$$

$$\mathcal{R}_{\{4,4\}} \quad -\mathrm{Sin}[\theta]^2\left(-1 + 2e^{2(\beta[r]+\gamma[r])}\beta'[r]^2 + 2e^{2(\beta[r]+\gamma[r])}\beta'[r]\gamma'[r] + e^{2(\beta[r]+\gamma[r])}\beta''[r]\right)$$

The Ricci Scalar from the Code

The General equations:

$$R = \mathcal{G}^{\mu\nu} R_{\mu\nu}$$

The Output of Mathematica Code

$$-2\, e^{-2\beta[r]} + 6\, e^{2\gamma[r]}\, \beta'[r]^2 + 8\, e^{2\gamma[r]}\, \beta'[r]\, \gamma'[r] + 4\, e^{2\gamma[r]}\, \gamma'[r]^2 + 4\, e^{2\gamma[r]}\, \beta''[r] + 2\, e^{2\gamma[r]}\, \gamma''[r]$$

The Einstein tensor from the Code

The General equations:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}\mathcal{G}_{\mu\nu}R$$

The Output of Mathematica Code

$$G_{\{1,1\}} \quad e^{-2\beta[r]+2\gamma[r]} - 3e^{4\gamma[r]}\beta'[r]^2 - 2e^{4\gamma[r]}\beta'[r]\gamma'[r] - 2e^{4\gamma[r]}\beta''[r]$$

$$G_{\{2,2\}} \quad -e^{-2(\beta[r]+\gamma[r])} + \beta'[r]^2 + 2\beta'[r]\gamma'[r]$$

$$G_{\{3,3\}} \quad e^{2(\beta[r]+\gamma[r])}\left(\beta'[r]^2 + 2\beta'[r]\gamma'[r] + 2\gamma'[r]^2 + \beta''[r] + \gamma''[r]\right)$$

$$G_{\{4,4\}} \quad e^{2(\beta[r]+\gamma[r])}\,\mathrm{Sin}[\theta]^2\left(\beta'[r]^2 + 2\beta'[r]\gamma'[r] + 2\gamma'[r]^2 + \beta''[r] + \gamma''[r]\right)$$

The Einstein Field equations from the Code

The General equations:

$$G_{\mu\nu} - \kappa T_{\mu\nu} = 0$$

The Output of Mathematica Code

$$\text{Eqn}_{\{1,1\}} \quad e^{-2\beta[r]+2\gamma[r]} - \mathcal{K}\,\mathcal{T}_o[r] - 3\,e^{4\gamma[r]}\,\beta'[r]^2 - 2\,e^{4\gamma[r]}\,\beta'[r]\,\gamma'[r] - 2\,e^{4\gamma[r]}\,\beta''[r]$$

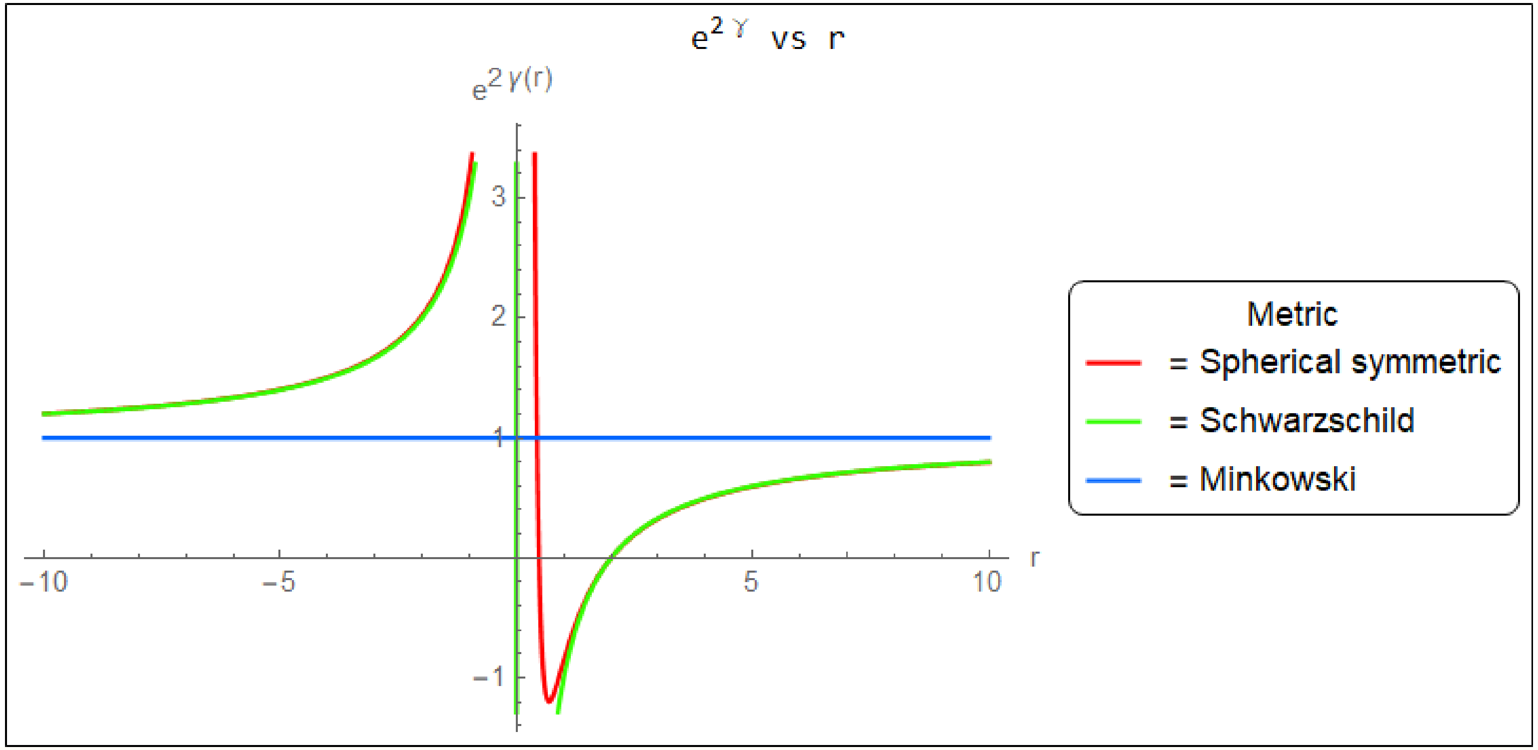
$$\text{Eqn}_{\{2,2\}} \quad -e^{-2(\beta[r]+\gamma[r])} - \mathcal{K}\,\mathcal{T}_1[r] + \beta'[r]^2 + 2\,\beta'[r]\,\gamma'[r]$$

$$\text{Eqn}_{\{3,3\}} \quad -\mathcal{K}\,\mathcal{T}_2[r] + e^{2(\beta[r]+\gamma[r])}\left(\beta'[r]^2 + 2\,\beta'[r]\,\gamma'[r] + 2\,\gamma'[r]^2 + \beta''[r] + \gamma''[r]\right)$$

$$\text{Eqn}_{\{4,4\}} \quad -\mathcal{K}\,\mathcal{T}_3[r] + e^{2(\beta[r]+\gamma[r])}\,\text{Sin}[\theta]^2\left(\beta'[r]^2 + 2\,\beta'[r]\,\gamma'[r] + 2\,\gamma'[r]^2 + \beta''[r] + \gamma''[r]\right)$$

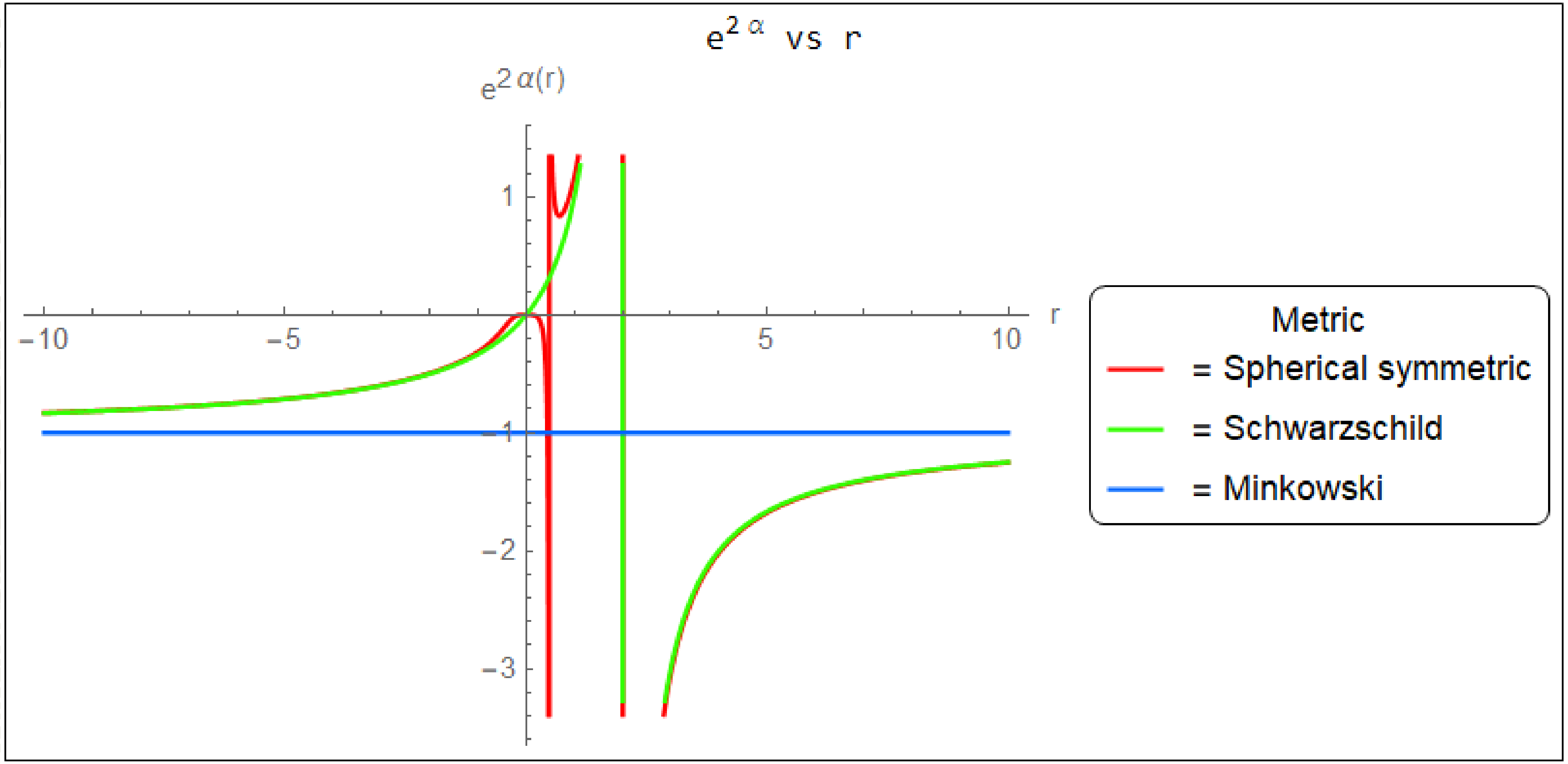
The γ graphs from the Code

$$\left\{\gamma[r] \to \frac{1}{2}\mathrm{Log}\left[-2\left(-\frac{1}{2} - \frac{1}{12r^4} + \frac{97}{96r}\right)\right]\right\}$$



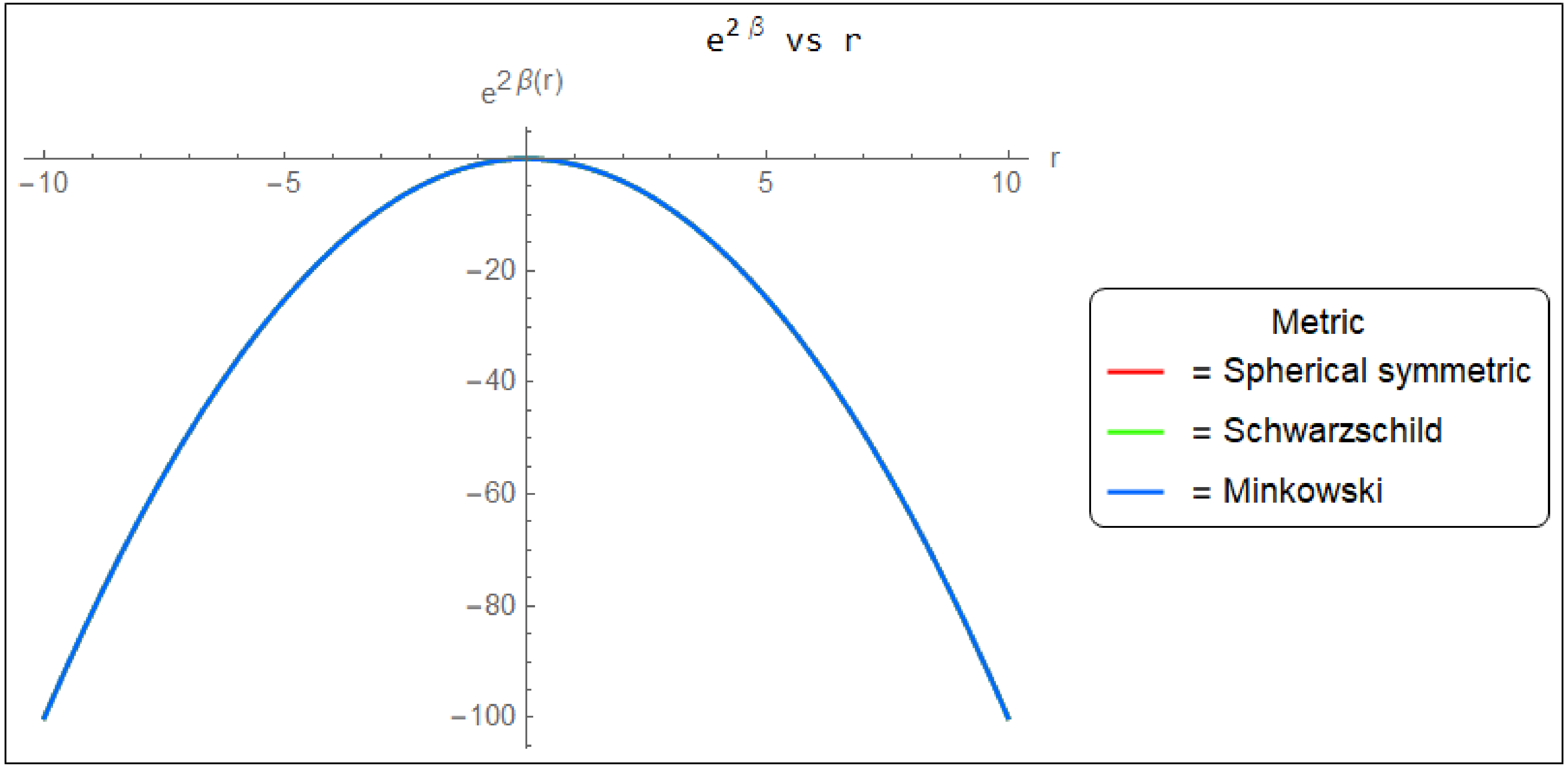
The α graphs from the Code

$$\{\alpha[r] \to -\gamma[r]\}$$

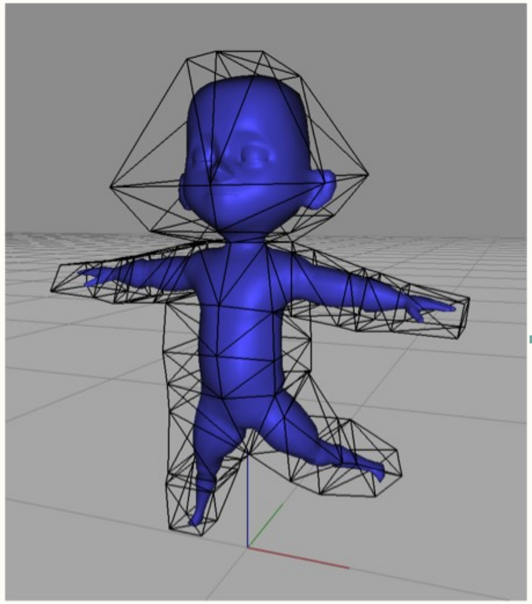


The β graphs from the Code

$\{\beta[r] \to \mathrm{Log}[r]\}$



Harmonic coordinates



The General equations:

$$\alpha = \gamma + 2\,\beta$$

In Riemannian geometry, a branch of mathematics, harmonic coordinates are a certain kind of coordinate chart on a smooth manifold, determined by a Riemannian metric on the manifold.

The well known property of the harmonic coordinates is that the covariant divergence of a vector field and the d'Alambertian of a scalar field take a particularly simple form:

$$D_\mu A^\mu \to g^{\mu\nu}\partial_\mu A_\nu,$$
$$g^{\mu\nu}D_\nu D_\mu\phi \to g^{\mu\nu}\partial_\mu\partial_\nu\phi.$$

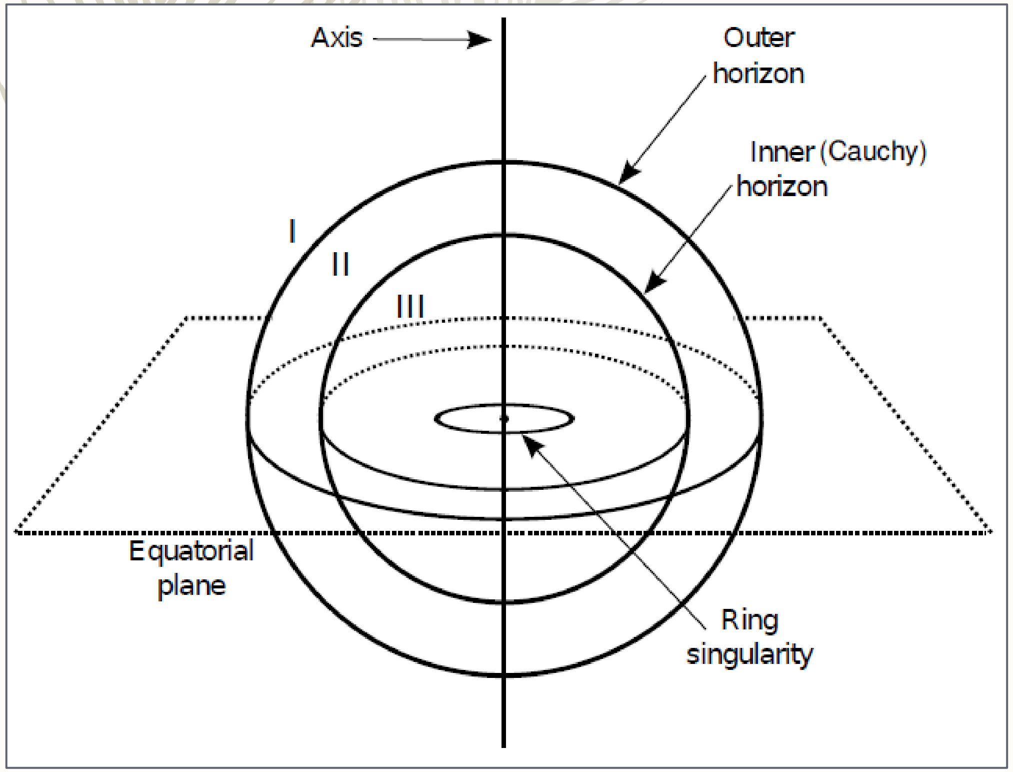
The harmonic condition

$$\partial_\mu\left(\sqrt{-g}\,g^{\mu\nu}\right) = 0 \qquad (1)$$

General Theory of Relativity

The spherical symmetric metric in harmonic coordinates:

$$ds^2 = e^{2\gamma(r)}dt^2 - e^{2(\gamma+2\,\beta)}dr^2 - e^{2\beta(r)}(d\theta^2 + \sin\theta^2\, d\phi^2)$$



$$\begin{pmatrix} e^{2\gamma[r]} & 0 & 0 & 0 \\ 0 & -e^{-2(2\beta[r]+\gamma[r])} & 0 & 0 \\ 0 & 0 & -e^{2\beta[r]} & 0 \\ 0 & 0 & 0 & -e^{2\beta[r]}\mathrm{Sin}[\theta]^2 \end{pmatrix}$$

The Christoffel symbols from the Code

The General equations:

$$\Gamma^{\lambda}{}_{\mu\nu} = \frac{1}{2}\mathcal{G}^{\lambda i}\left(\frac{\partial \mathcal{G}_{i\mu}}{\partial \chi^{\nu}} + \frac{\partial \mathcal{G}_{i\nu}}{\partial \chi^{\mu}} - \frac{\partial \mathcal{G}_{\mu\nu}}{\partial \chi^{i}}\right)$$

The Output of Mathematica Code

$\Gamma^{1}{}_{\{2,1\}}$	$\gamma'[r]$
$\Gamma^{2}{}_{\{1,1\}}$	$e^{4(\beta[r]+\gamma[r])}\gamma'[r]$
$\Gamma^{2}{}_{\{2,2\}}$	$-2\beta'[r]-\gamma'[r]$
$\Gamma^{2}{}_{\{3,3\}}$	$-e^{6\beta[r]+2\gamma[r]}\beta'[r]$
$\Gamma^{2}{}_{\{4,4\}}$	$-e^{6\beta[r]+2\gamma[r]}\mathrm{Sin}[\theta]^{2}\beta'[r]$
$\Gamma^{3}{}_{\{3,2\}}$	$\beta'[r]$
$\Gamma^{3}{}_{\{4,4\}}$	$-\mathrm{Cos}[\theta]\,\mathrm{Sin}[\theta]$
$\Gamma^{4}{}_{\{4,2\}}$	$\beta'[r]$
$\Gamma^{4}{}_{\{4,3\}}$	$\mathrm{Cot}[\theta]$

The Riemann tensor from the Code

The General equations:

$$R^{\lambda}{}_{\omega\nu\mu} = \frac{\partial\Gamma^{\lambda}{}_{\mu\omega}}{\partial\chi^{\nu}} - \frac{\partial\Gamma^{\lambda}{}_{\mu\nu}}{\partial\chi^{\omega}} + \sum_{s}\left(\Gamma^{s}{}_{\mu\omega}\Gamma^{\lambda}{}_{\nu s} - \Gamma^{s}{}_{\mu\nu}\Gamma^{\lambda}{}_{\omega s}\right)$$

The Output of Mathematica Code

$R^1{}_{\{2,2,1\}}$	$2\,\beta'[r]\,\gamma'[r] + 2\,\gamma'[r]^2 + \gamma''[r]$
$R^1{}_{\{3,3,1\}}$	$e^{6\beta[r]+2\gamma[r]}\,\beta'[r]\,\gamma'[r]$
$R^1{}_{\{4,4,1\}}$	$e^{6\beta[r]+2\gamma[r]}\,\mathrm{Sin}[\theta]^2\,\beta'[r]\,\gamma'[r]$
$R^2{}_{\{1,2,1\}}$	$e^{4(\beta[r]+\gamma[r])}\left(2\,\beta'[r]\,\gamma'[r] + 2\,\gamma'[r]^2 + \gamma''[r]\right)$
$R^2{}_{\{3,3,2\}}$	$e^{6\beta[r]+2\gamma[r]}\left(3\,\beta'[r]^2 + \beta'[r]\,\gamma'[r] + \beta''[r]\right)$
$R^2{}_{\{4,4,2\}}$	$e^{6\beta[r]+2\gamma[r]}\,\mathrm{Sin}[\theta]^2\left(3\,\beta'[r]^2 + \beta'[r]\,\gamma'[r] + \beta''[r]\right)$
$R^3{}_{\{1,3,1\}}$	$e^{4(\beta[r]+\gamma[r])}\,\beta'[r]\,\gamma'[r]$
$R^3{}_{\{2,3,2\}}$	$-3\,\beta'[r]^2 - \beta'[r]\,\gamma'[r] - \beta''[r]$
$R^3{}_{\{4,4,3\}}$	$\mathrm{Sin}[\theta]^2\left(-1 + e^{6\beta[r]+2\gamma[r]}\,\beta'[r]^2\right)$
$R^4{}_{\{1,4,1\}}$	$e^{4(\beta[r]+\gamma[r])}\,\beta'[r]\,\gamma'[r]$
$R^4{}_{\{2,4,2\}}$	$-3\,\beta'[r]^2 - \beta'[r]\,\gamma'[r] - \beta''[r]$
$R^4{}_{\{3,4,3\}}$	$1 - e^{6\beta[r]+2\gamma[r]}\,\beta'[r]^2$

The Ricci tensor from the Code

The General equations:

$$R_{\omega\mu} = R^{\lambda}{}_{\omega\lambda\mu}$$

The Output of Mathematica Code

$$\mathcal{R}_{\{1,1\}} \quad e^{4(\beta[r]+\gamma[r])}\left(4\,\beta'[r]\,\gamma'[r] + 2\,\gamma'[r]^2 + \gamma''[r]\right)$$

$$\mathcal{R}_{\{2,2\}} \quad -6\,\beta'[r]^2 - 4\,\beta'[r]\,\gamma'[r] - 2\,\gamma'[r]^2 - 2\,\beta''[r] - \gamma''[r]$$

$$\mathcal{R}_{\{3,3\}} \quad 1 - 4\,e^{6\beta[r]+2\gamma[r]}\,\beta'[r]^2 - 2\,e^{6\beta[r]+2\gamma[r]}\,\beta'[r]\,\gamma'[r] - e^{6\beta[r]+2\gamma[r]}\,\beta''[r]$$

$$\mathcal{R}_{\{4,4\}} \quad -\mathrm{Sin}[\theta]^2\left(-1 + 4\,e^{6\beta[r]+2\gamma[r]}\,\beta'[r]^2 + 2\,e^{6\beta[r]+2\gamma[r]}\,\beta'[r]\,\gamma'[r] + e^{6\beta[r]+2\gamma[r]}\,\beta''[r]\right)$$

The Einstein tensor from the Code

The General equations:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}\mathcal{G}_{\mu\nu}R$$

The Output of Mathematica Code

$G_{\{1,1\}}$	$e^{-2\beta[r]+2\gamma[r]} - 7\, e^{4(\beta[r]+\gamma[r])}\,\beta'[r]^2 - 2\, e^{4(\beta[r]+\gamma[r])}\,\beta'[r]\,\gamma'[r] - 2\, e^{4(\beta[r]+\gamma[r])}\,\beta''[r]$
$G_{\{2,2\}}$	$-e^{-2(3\beta[r]+\gamma[r])} + \beta'[r]^2 + 2\,\beta'[r]\,\gamma'[r]$
$G_{\{3,3\}}$	$e^{6\beta[r]+2\gamma[r]}\left(3\,\beta'[r]^2 + 4\,\beta'[r]\,\gamma'[r] + 2\,\gamma'[r]^2 + \beta''[r] + \gamma''[r]\right)$
$G_{\{4,4\}}$	$e^{6\beta[r]+2\gamma[r]}\,\mathrm{Sin}[\theta]^2\left(3\,\beta'[r]^2 + 4\,\beta'[r]\,\gamma'[r] + 2\,\gamma'[r]^2 + \beta''[r] + \gamma''[r]\right)$

The Einstein Field equations from the Code

The General equations:

$$G_{\mu\nu} - \kappa T_{\mu\nu} = 0$$

The Output of Mathematica Code

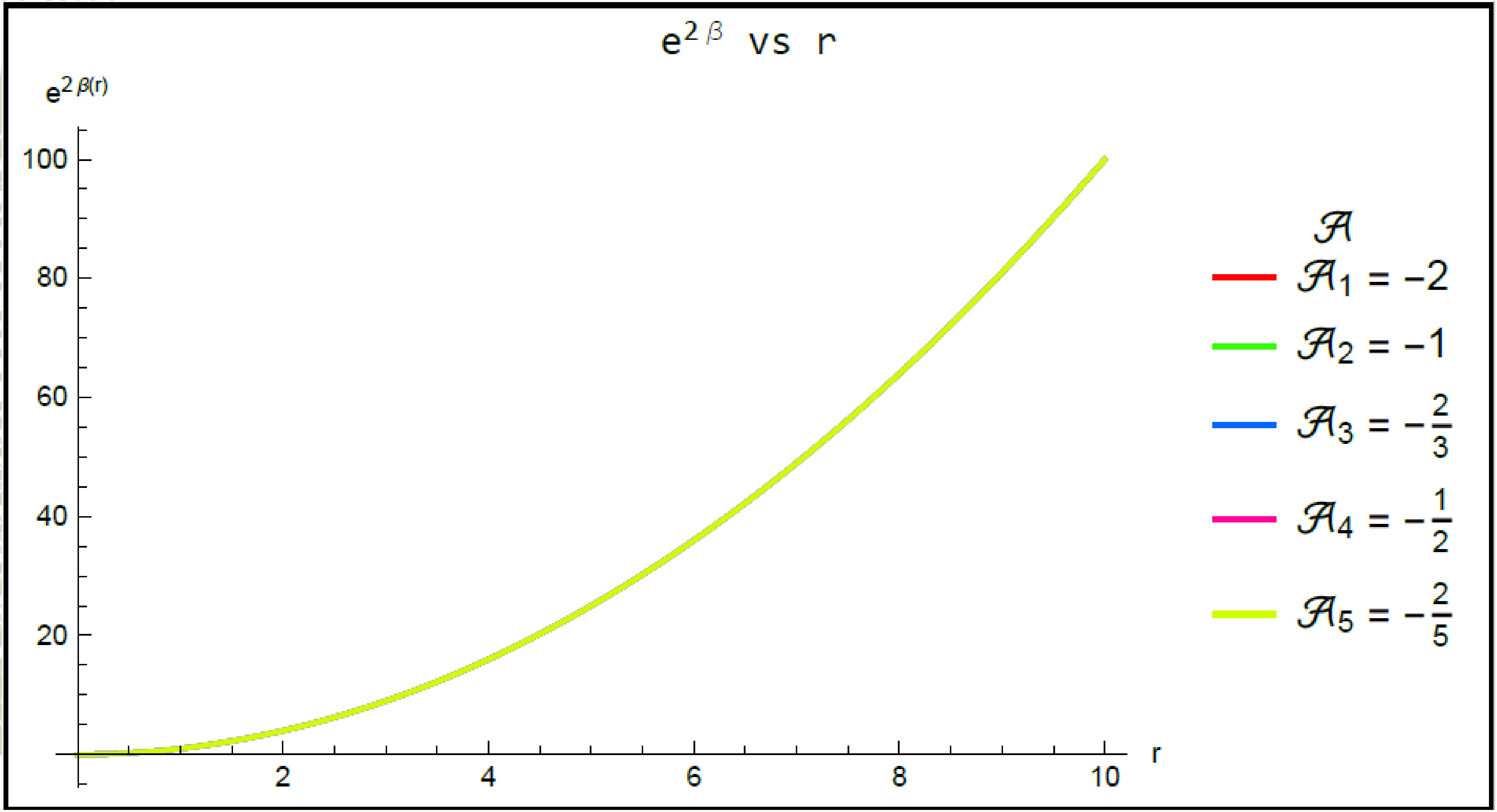
$$\{G_{\{1,1\}}, -, T_{\{1,1\}}\} \quad e^{-2\beta[r]+2\gamma[r]} - \mathcal{K}\mathcal{T}_0[r] - 7\,e^{4(\beta[r]+\gamma[r])}\,\beta'[r]^2 - 2\,e^{4(\beta[r]+\gamma[r])}\,\beta'[r]\,\gamma'[r] - 2\,e^{4(\beta[r]+\gamma[r])}\,\beta''[r]$$

$$\{G_{\{2,2\}}, -, T_{\{2,2\}}\} \quad -e^{-2(3\beta[r]+\gamma[r])} - \mathcal{K}\mathcal{T}_1[r] + \beta'[r]^2 + 2\,\beta'[r]\,\gamma'[r]$$

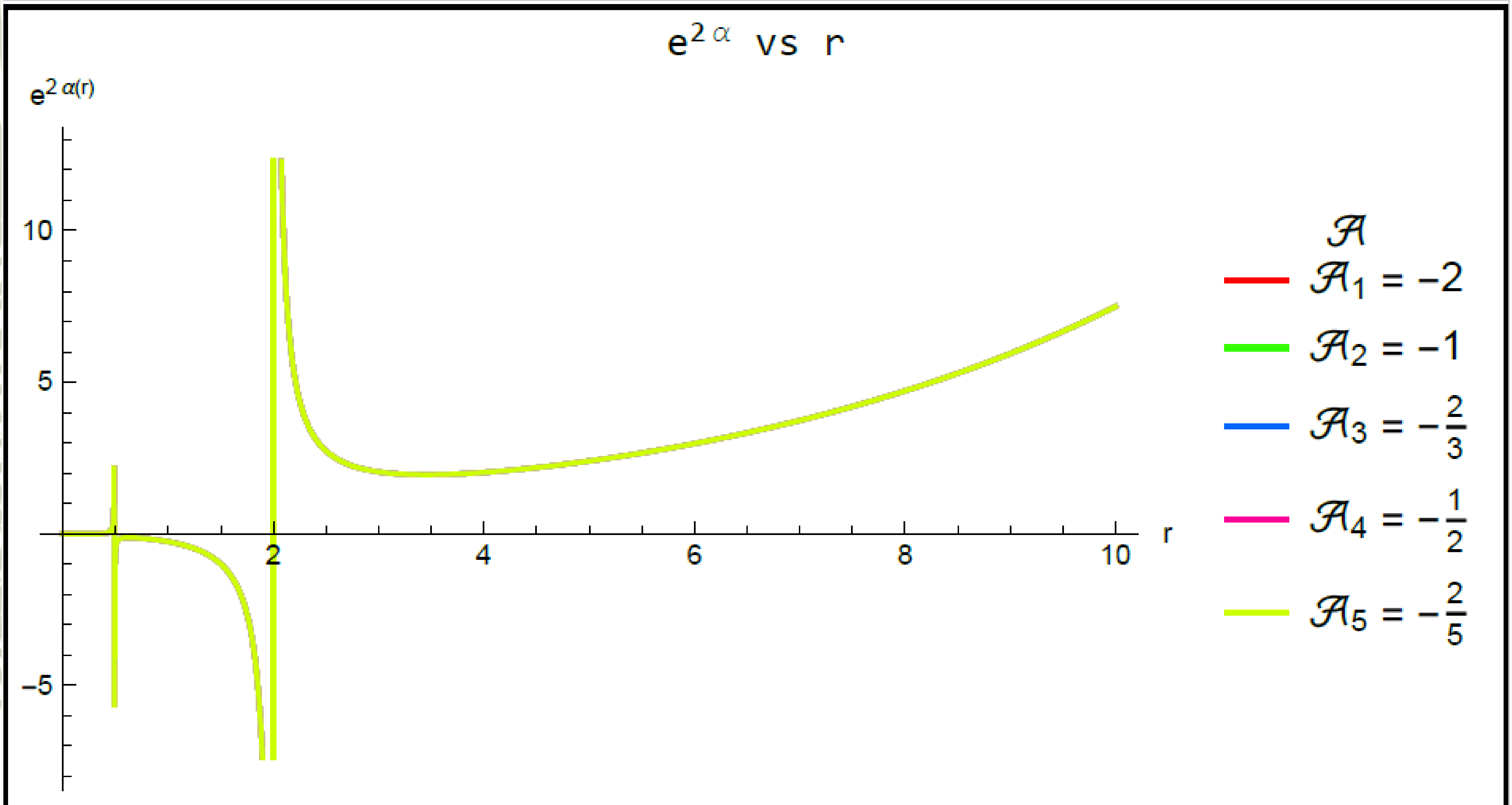
$$\{G_{\{3,3\}}, -, T_{\{3,3\}}\} \quad -\mathcal{K}\mathcal{T}_2[r] + e^{6\beta[r]+2\gamma[r]}\left(3\,\beta'[r]^2 + 4\,\beta'[r]\,\gamma'[r] + 2\,\gamma'[r]^2 + \beta''[r] + \gamma''[r]\right)$$

$$\{G_{\{4,4\}}, -, T_{\{4,4\}}\} \quad -\mathcal{K}\mathcal{T}_3[r] + e^{6\beta[r]+2\gamma[r]}\,\mathrm{Sin}[\theta]^2\left(3\,\beta'[r]^2 + 4\,\beta'[r]\,\gamma'[r] + 2\,\gamma'[r]^2 + \beta''[r] + \gamma''[r]\right)$$

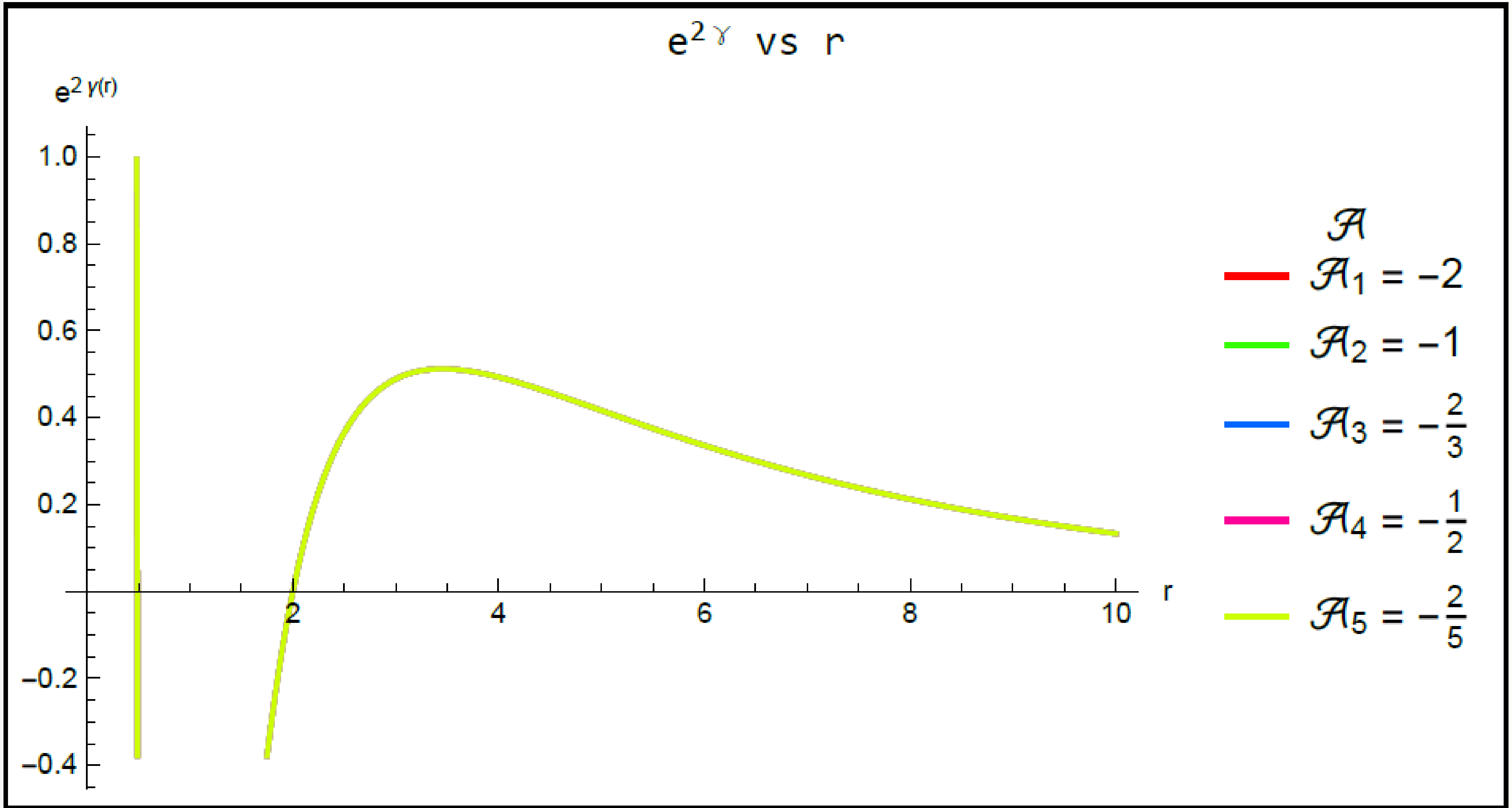
Graph of beta with r of GR in Spherical coordinates



Graph of alpha with r of GR in Spherical coordinates



Graph of gamma with r of GR in Spherical coordinates



Summarize and future work

- ***We build Mathematica code that can calculate, represent (equation and graphically) and solve the Einstein field equations that very important in the theory of general relativity.***
- ***The case study we use show a very match results old models.***
- ***We hope that in future we can describe the core of very heavy astronomical objects like black hole (in cooperation with nuclear structure group).***
- ***Also we hope to use this model with quantum information principles to describe how black hole communicate with its surrounding.***
- ***Making mathematical modeling for electromagnetic wave propagation inside some stars and we will try to analyze some affects on the Earth.***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