Elastic scattering using the optical model within NRV Knowledge Base

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Aims of the project

To understand:

1. Mathematical formulation of low-energy quantum elastic scattering
2. Use NRV Knowledge Base in describing elastic scattering
3. How to describe elastic scattering in NRV:
    - $^{4}\mathrm{He} + {}^{58}\mathrm{Ni} \rightarrow {}^{4}\mathrm{He} + {}^{58}\mathrm{Ni}$
    - $^{4}\mathrm{He} + {}^{209}\mathrm{Bi} \rightarrow {}^{4}\mathrm{He} + {}^{209}\mathrm{Bi}$

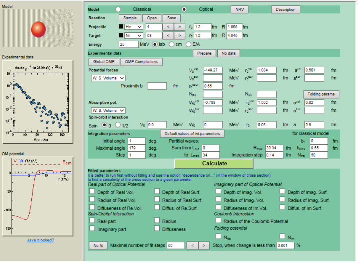
    at different energies

Introduction: Nuclear Reactions Video (NRV) Knowledge Base

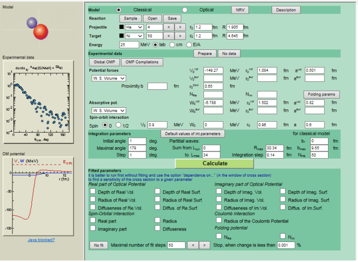
1. NRV: Developed at Flerov Laboratory of Nuclear Reactions, Joint Institute for Nuclear Research
2. Allows one to simulate many channels of nuclear reactions at low energies: Elastic scattering, inelastic scattering, transfer reactions, fusion, etc.

Figure: NRV homepage. URL: http://nrv.jinr.ru

3. In this project: elastic scattering of nuclei within the NRV project.



(a) Input page, Quantum OM



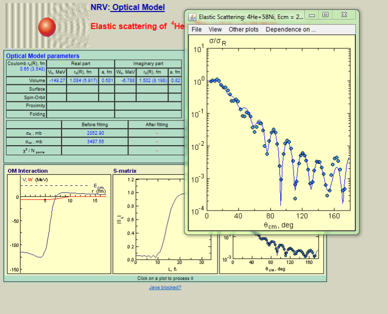
(b) Input page, Classical.

Figure

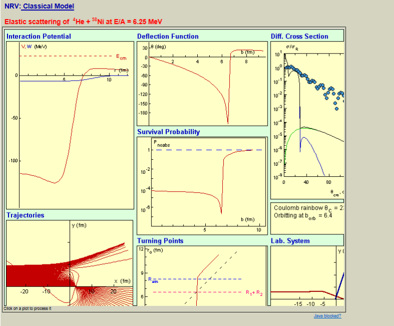
Input information

1. Reaction:
   - The projectile and target selected from the drop-down lists
   - Mass number of the projectile and target
   - Energy (lab., centre-of-mass or energy per nucleon of projectile)
2. Experimental data, if available. May be copied into NRV from a text file
3. Potential: A real and imaginary part of the optical potential; spin-orbit potential, if the projectile has a spin. Automatic values available for potential parameters.
4. Integration parameters

NRV results page



(a) Results page, OM: : σ, $d\sigma/d\Omega$, $S_\ell(k)$, partial wavefunctions, etc



(b) Results page, Classical: σ, $d\sigma/d\Omega$, trajectories field, deflection, turning point, survival prob., etc

Figure

1. Elastic scattering: $A + B \to A + B$
2. Time-independent formalism: stationary states
3. Time-Independent Schrödinger Equation: $H\psi^+ = E\psi^+$, $E > 0$
4. Boundary conditions
5. Spherically symmetric domain, total wavefunction $\psi(r, \theta, \phi)$ separable: $\psi(r, \theta, \phi) = \underbrace{(\psi_\ell(r)/r)}_{radial} \underbrace{Y_\ell^m(\theta, \phi)}_{spherical}$
6. Azimuthal symmetry, $m = 0$, $Y_\ell^0(\theta, \phi) = \sqrt{(2\ell + 1)/4\pi} P_\ell(\cos\theta)$
7. $\psi_\ell(r)$ satisfies the single-channel Radial Schrödinger Equation:

$$-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}\psi_\ell + \left[V(r) + \frac{\ell(\ell+1)\hbar^2}{2\mu r^2}\right]\psi_\ell = E_{cm}\psi_\ell \tag{1}$$

$V(r)$ is the potential.

1. At $r = 0$

$$\psi^+_{k\ell}(r) = 0$$

2. Asymptotic region (Outgoing waves boundary conditions)

$$\psi^+_{k\ell}(r) \to \frac{i}{2}[H^-_\ell(kr;\eta) - S_\ell(k)H^+_\ell(kr;\eta)], \quad r \to \infty \tag{2}$$

   where $S_\ell(k) = e^{2i(\sigma_\ell(k)+\delta^N_\ell(k))}$, η the Sommerfeld parameter, $H^-_\ell(kr;\eta)$ and $H^+_\ell(kr;\eta)$ are the incoming and outgoing waves, respectively.

3. $H^\pm_\ell(kr;\eta) = G_\ell(kr;\eta) \pm iF_\ell(kr;\eta)$,
   where $G_\ell(kr;\eta)$ and $F_\ell(kr;\eta)$ are the regular and irregular Coulomb functions, resp.; two linearly independent solutions of pure Coulomb scattering.

4. Numerov Method is used in NRV to solve the Radial Schrödinger Equation

1. Partial wave analysis (Faxen-Holzmark formalism):

$$f(\theta) = \frac{1}{2ik}\sum_{\ell=0}^{\infty}(S_\ell(k) - 1)[(2\ell + 1)P_\ell(\cos\theta)] \quad (3)$$

   $S_\ell(k)$ contains information on nature of the interaction: short range, long range
2. $f(\theta) = f_C(\theta) + f_N(\theta)$
3. $f(\theta)$ establishes the link between scattering theory and experiments.

   Differetial cross-section: (4)

$$\frac{d\sigma}{d\Omega} = |f(\theta, \phi)|^2 \quad (5)$$

   Integrated cross-section: (6)

$$\sigma = 4\pi \sum_{\ell=0}^{\infty}(2\ell + 1)|f_\ell|^2 \quad (7)$$

   where f_ℓ are the partial scattering amplitudes.

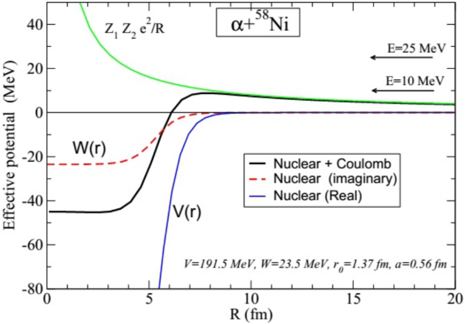
1. $V(r) = \underbrace{V_C(r)}_{Coulomb} + \underbrace{V_N(r) + iW_N(r)}_{Nuclear} + \underbrace{V_{SO}(r) + iW_{SO}(r)(\vec{\ell}.\vec{s})}_{Spin-orbit}$
2. Coulomb potential for uniform charge distribution:

$$V_C(r) = \begin{cases} Z_P Z_T e^2 . \frac{1}{2R_C} . (3 - \frac{r^2}{R_C^2}), & \text{for} \quad r < R_C \\ Z_P Z_T e^2 . \frac{1}{r}, & \text{for} \quad r > R_C \end{cases} \tag{8}$$

3. $V_N(r)$ and $W_N(r)$ may be approximated as follows:
   - Woods-Saxon Volume (WSV) potential
   - Woods-Saxon Surface (WSS) potential
   - Superposition of WSV and WSS
   - Folding Potential

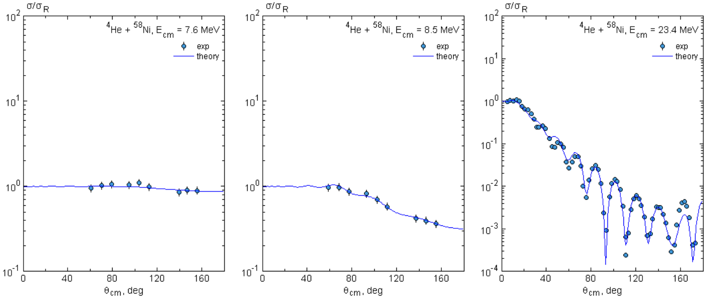
1. In this project WSV potentials are used for both $V_N(r)$ and $W_N(r)$:

$$V_N(r) = -\frac{V_o}{1 + exp[(r - r_v)/a_v]}, \quad W_N(r) = -\frac{W_o}{1 + exp[(r - r_w)/a_w]}$$



1. Two simulations: alpha particle + heavy-ion
   ^{4}He has zero spin, no spin-orbit interaction, $V_{SO} = W_{SO} = 0$
   - $$^4\text{He} + {}^{58}\text{Ni} \rightarrow {}^4\text{He} + {}^{58}\text{Ni}$$
     $$\text{Coulomb barrier} = 11.50\,\text{MeV}(c.m.)$$
     $$E_{cm} = 7.6,\ 8.5,\ \text{and}\ 23.4\,\text{MeV}$$
   - $$^4\text{He} + {}^{209}\text{Bi} \rightarrow {}^4\text{He} + {}^{209}\text{Bi}$$
     $$\text{Coulomb barrier} = 26.00\,\text{MeV}(c.m.)$$
     $$E_{cm} = 24.3,\ 34.0,\ \text{and}\ 68.2\,\text{MeV}$$

$^4He + {}^{58}Ni \rightarrow {}^4He + {}^{58}Ni$



(a) $\eta = 6.196$ (b) $\eta = 5.846$ (c) $\eta = 3.527$

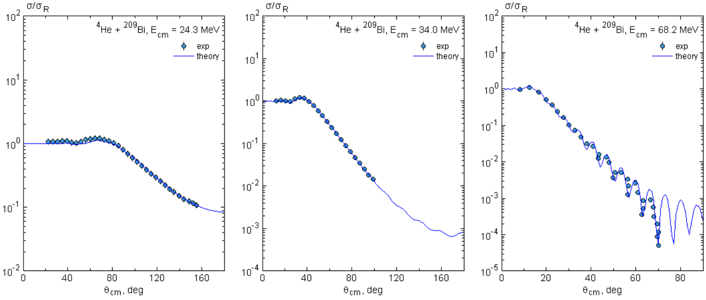
Figure: Ratio of total cross-section to Rutherford cross-section. Rutherford scattering at low energies; Fresnel behaviour around barrier, Fraunhofer oscillations for high energies beyong Coulomb barrier.

$\eta = \frac{Z_P Z_T e^2}{\hbar}(\frac{\mu}{2E})^{1/2}$; Nickel-58, Z=28

	Real (Woods-Saxon Vol.)			Imaginary (Woods-Saxon Vol.)		
E_{cm}/MeV	V_o/MeV	r_v/fm	a_v/fm	W_o	r_w/fm	a_w/fm
7.6	-155.106	1.068	0.53	-14.886	0.479	0.820
8.5	-154.134	1.027	0.676	-16.866	0.306	1.649
23.4	-149.270	1.084	0.501	-6.788	1.502	0.820

Table: Wood-Saxon Vol. parameters for the nucleus-nucleus potential

^{4}He + ^{209}Bi → ^{4}He + ^{209}Bi



(a) $\eta = 10.497$ (b) $\eta = 8.874$ (c) $\eta = 6.270$

Figure: Ratio of total cross-section to Rutherford cross-section. Fresnel behaviour around barrier, Fraunhofer oscillations for high energies beyong Coulomb barrier.

$\eta = \frac{Z_P Z_T e^2}{\hbar}\left(\frac{\mu}{2E}\right)^{1/2}$; Bismuth-209, Z= 83

	Real (Woods-Saxon Vol.)			Imaginary (Woods-Saxon Vol.)		
E_{cm}/MeV	V_o/MeV	r_v/fm	a_v/fm	W_o	r_w/fm	a_w/fm
24.3	-162.9	0.966	0.656	-11.478	0.979	0.472
34.0	-163.0	1.035	0.593	-17.128	1.134	0.400
68.2	-163.5	1.014	0.656	-17.800	1.153	0.600

Table: Wood-Saxon Vol. parameters for the nucleus-nucleus potential

Conclusion

1. Elastic scattering studied within the Optical Model
2. NRV OM code used to study
   $^{4}He + {}^{58}Ni$
   $^{4}He + {}^{209}Bi$
   at different energies
3. We have obtained a good agreement between the calculations and experimental data