Crystal structures in generalized Skyrme model

Ilya Perapechka

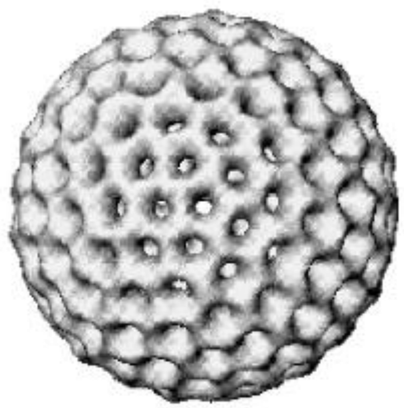
Department of Theoretical Physics and Astrophysics

Belarusian State University

Project supervisor: Prof. Yakov Shnir

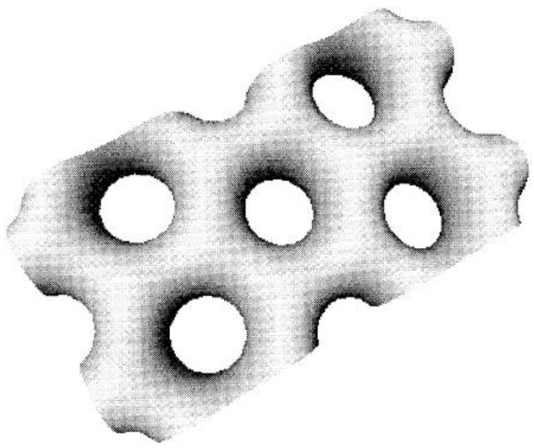
Phys. Rev. D 96, 045013 (2017)

Skyrme model: history

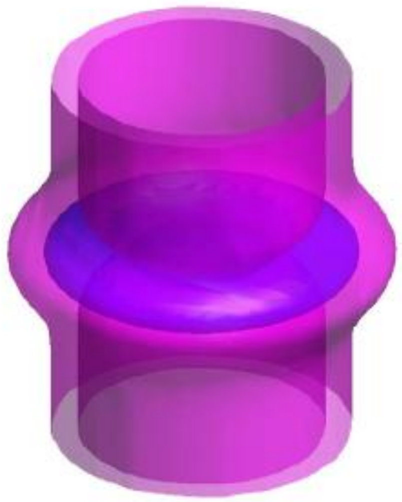


$B = 97$ Y-symmetric skyrmion

Chain of two $B = 2$ skyrmion-antiskyrmion pairs

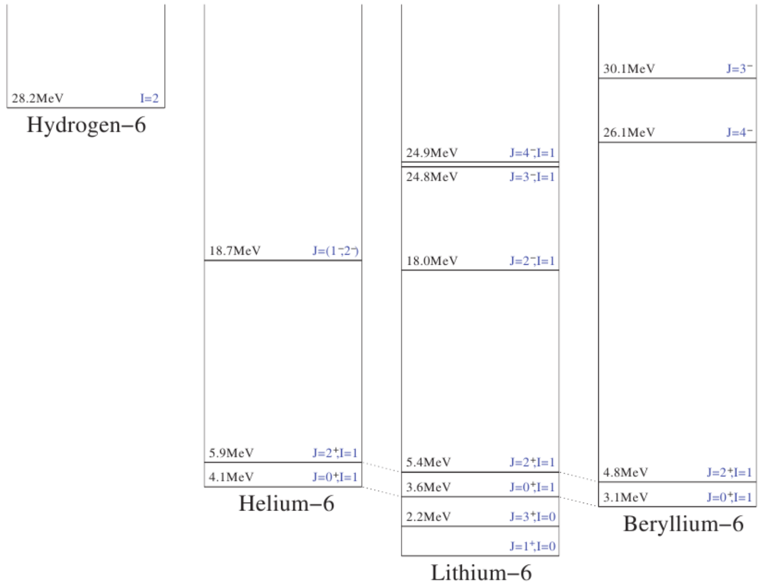


Hexagonal Skyrme lattice

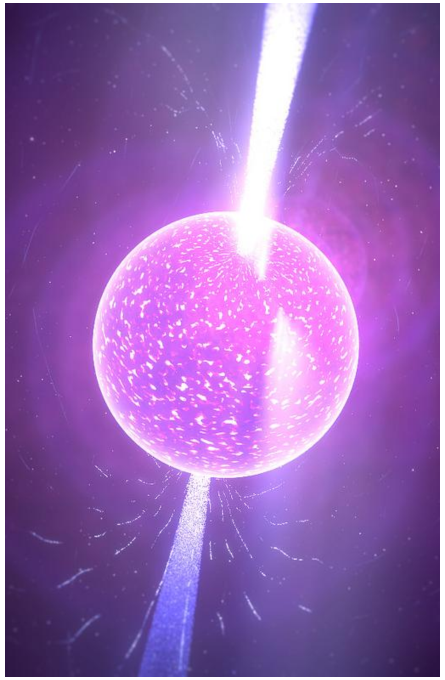


Vortex with a trapped skyrmion

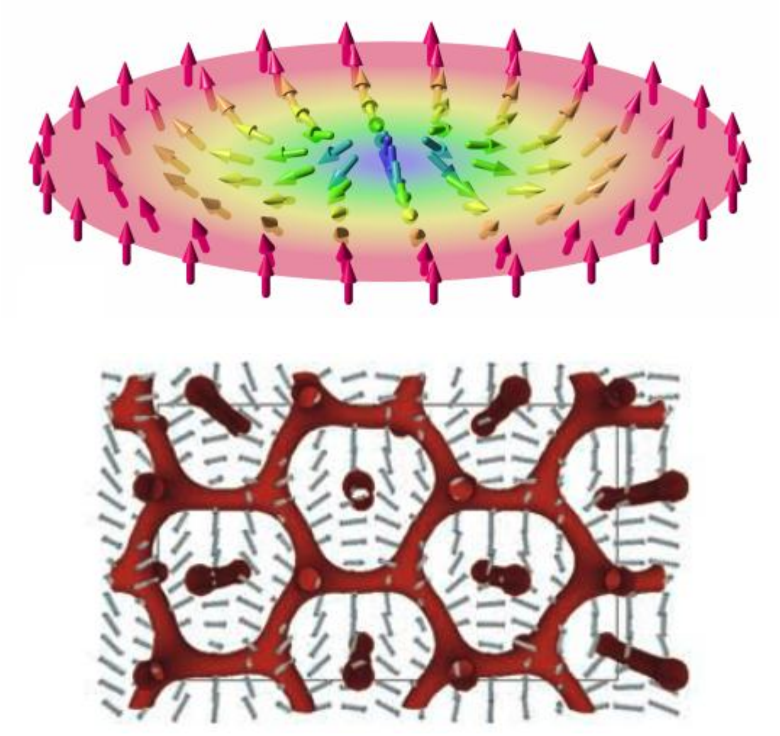
Skyrme model: applications



Effective theory for nuclear and particle physics



Neutron stars matter



Quasiparticles in condensed matter

Generalized Skyrme model

Skyrme field: $U(\boldsymbol{x}) \in \mathrm{SU}(2) \xrightarrow[\boldsymbol{x}\to\infty]{} \mathbb{1} \Rightarrow U: S^3 \to S^3$

Topological charge: $B = \pi_3(S^3) = \frac{1}{24\pi^2}\int \varepsilon^{ijk}\mathrm{Tr}\left[\left(U^\dagger\partial_i U\right)\left(U^\dagger\partial_j U\right)\left(U^\dagger\partial_k U\right)\right]d^3x$

Left $\mathfrak{su}(2)$-current: $L_\mu = U^\dagger\partial_\mu U \Rightarrow B = \frac{1}{24\pi^2}\int \varepsilon^{ijk}\,\mathrm{Tr}\left[L_i L_j L_k\right]d^3x$

Topological current: $B^\mu = \frac{1}{24\pi^2}\varepsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\left(L_\nu L_\rho L_\sigma\right)$

$$\mathcal{L}_{0246} = \underbrace{\frac{a}{2}\mathrm{Tr}\left(L_\mu L^\mu\right)}_{\text{sigma-model term}} + \underbrace{\frac{b}{4}\mathrm{Tr}\left(\left[L_\mu, L_\nu\right]\left[L^\mu, L^\nu\right]\right)}_{\text{Skyrme term}} + \underbrace{4\pi^4 c B_\mu B^\mu}_{\text{sixth order term}} + \underbrace{m_\pi^2\mathcal{V}}_{\text{potential term}}$$

$$\mathcal{V} = \mathrm{Tr}\left(\frac{\mathbb{1}-U}{2}\right)$$

pion potential

$$\mathcal{V} = \mathrm{Tr}\left(\frac{\mathbb{1}-U}{2}\right)\mathrm{Tr}\left(\frac{\mathbb{1}+U}{2}\right)$$

double-vacuum potential

$$\mathcal{V} = \left(\mathrm{Tr}\left(\frac{\mathbb{1}-U}{2}\right)\right)^2\mathrm{Tr}\left(\frac{\mathbb{1}+U}{2}\right)$$

mixed potential

Generalized Skyrme model

Stress-energy tensor: $T_{\mu\nu} = a\mathrm{Tr}\left(L_\mu L_\nu - \frac{1}{2}\eta_{\mu\nu}L_\rho L^\rho\right) + b\mathrm{Tr}\left([L_\mu, L_\rho][L_\nu, L^\rho] - \frac{1}{4}\eta_{\mu\nu}[L_\rho, L_\sigma][L^\rho, L^\sigma]\right) +$
$8\pi^4 c\left(B_\mu B_\nu - \frac{1}{2}\eta_{\mu\nu}B_\rho B^\rho\right) - \eta_{\mu\nu}m_\pi^2\mathcal{V}$

$$\boldsymbol{n} = (\sigma, \boldsymbol{\pi}) \Rightarrow U = \sigma \cdot 1 + i\boldsymbol{\pi} \cdot \boldsymbol{\tau} \Rightarrow \sigma^2 + \boldsymbol{\pi} \cdot \boldsymbol{\pi} = 1$$

⇩

$$B = -\frac{1}{12\pi^2}\int \varepsilon_{abcd}\varepsilon^{ijk}n^a\partial_i n^b\partial_j n^c\partial_k n^d d^3x$$

$$E = \int \left\{a\partial_i\boldsymbol{n} \cdot \partial^i\boldsymbol{n} + 2b\left(\left(\partial_i\boldsymbol{n} \cdot \partial^i\boldsymbol{n}\right)^2 - \left(\partial_i\boldsymbol{n} \cdot \partial_j\boldsymbol{n}\right)\left(\partial^i\boldsymbol{n} \cdot \partial^j\boldsymbol{n}\right)\right) + c\left(\varepsilon^{abcd}\partial_1 n_a\partial_2 n_b\partial_3 n_c n_d\right)^2 + m_\pi^2\mathcal{V}\right\}d^3x$$

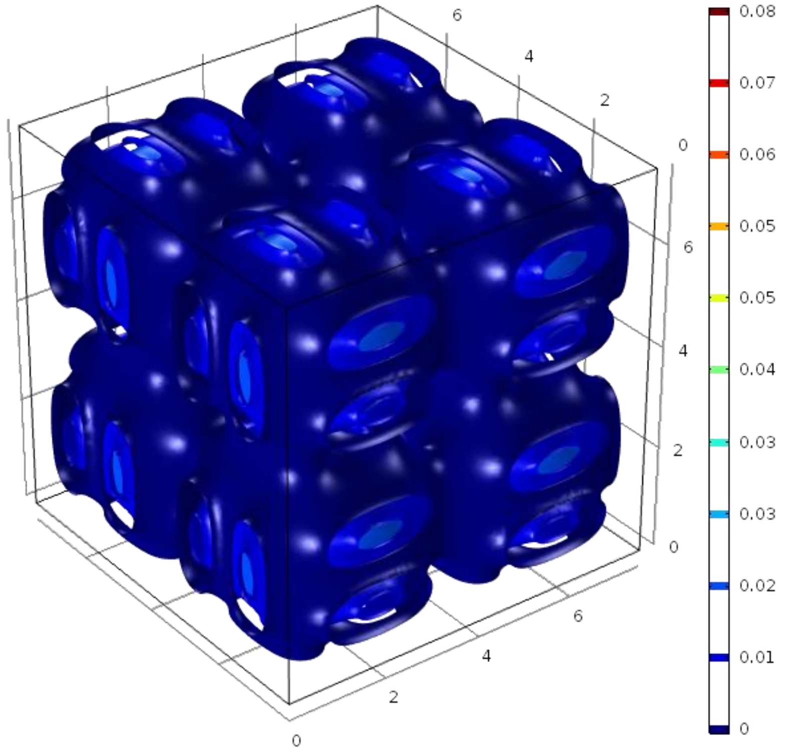
$$E \geq 24\pi^2|B|$$

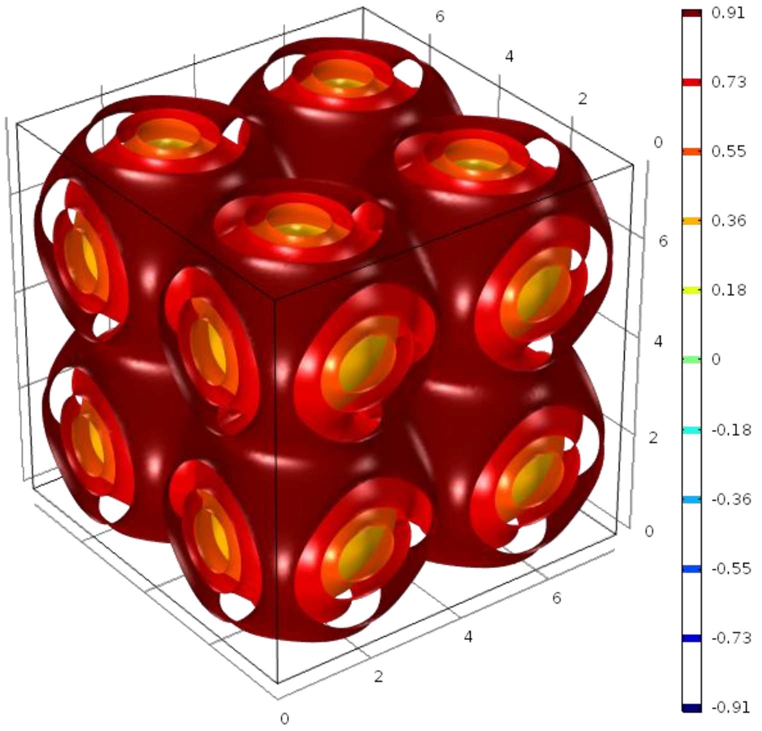
Klebanov's crystal

$$(x,y,z) \to (x+L,y,z) \Rightarrow (\sigma,\pi_1,\pi_2,\pi_3) \to (\sigma,-\pi_1,\pi_2,-\pi_3)$$

$$(x,y,z) \to (-x,y,z) \Rightarrow (\sigma,\pi_1,\pi_2,\pi_3) \to (\sigma,-\pi_1,\pi_2,\pi_3)$$

$$(x,y,z) \to (z,x,y) \Rightarrow (\sigma,\pi_1,\pi_2,\pi_3) \to (\sigma,\pi_3,\pi_1,\pi_2)$$





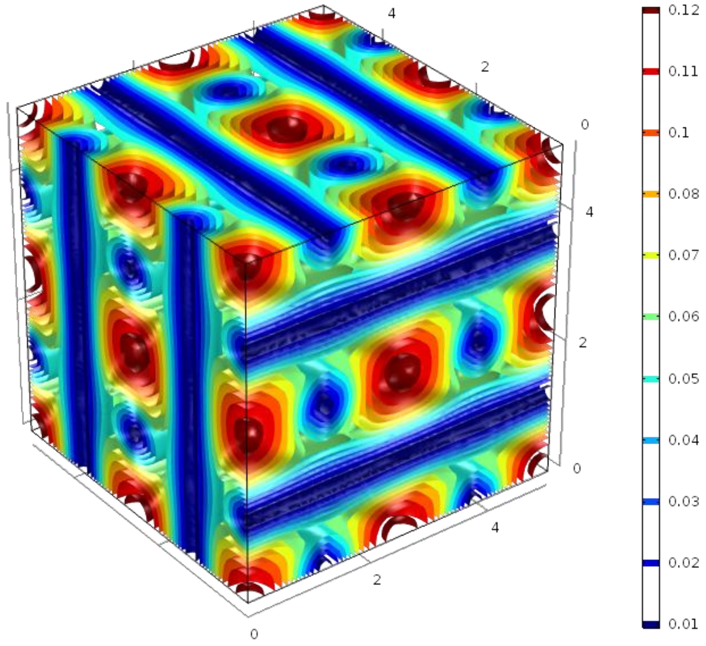
Body-centered cubic lattice

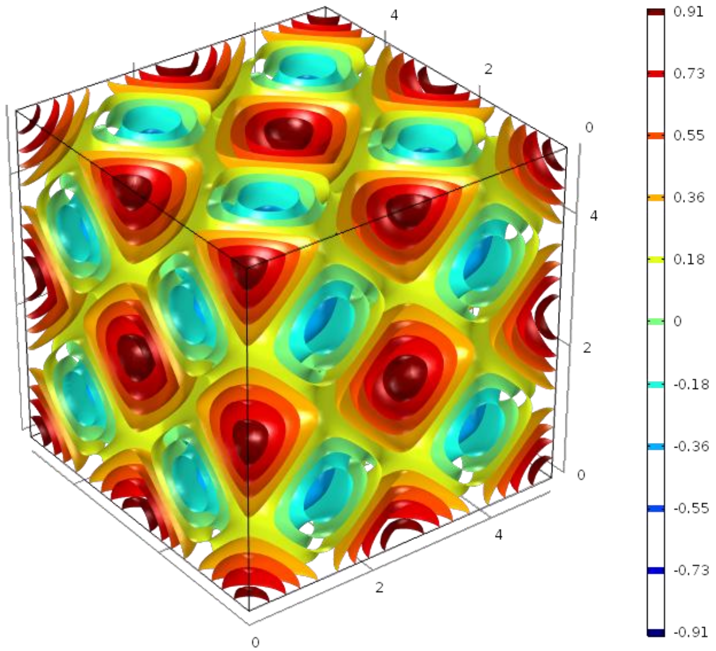
$$(x, y, z) \to (x + L, y, z) \Rightarrow (\sigma, \pi_1, \pi_2, \pi_3) \to (\sigma, -\pi_1, \pi_2, -\pi_3)$$

$$(x, y, z) \to (-x, y, z) \Rightarrow (\sigma, \pi_1, \pi_2, \pi_3) \to (\sigma, -\pi_1, \pi_2, \pi_3)$$

$$(x, y, z) \to (z, x, y) \Rightarrow (\sigma, \pi_1, \pi_2, \pi_3) \to (\sigma, \pi_3, \pi_1, \pi_2)$$

$$(x, y, z) \to \left(\frac{L}{2} - z, \frac{L}{2} - y, \frac{L}{2} - x\right) \Rightarrow (\sigma, \pi_1, \pi_2, \pi_3) \to (-\sigma, \pi_2, \pi_1, \pi_3)$$





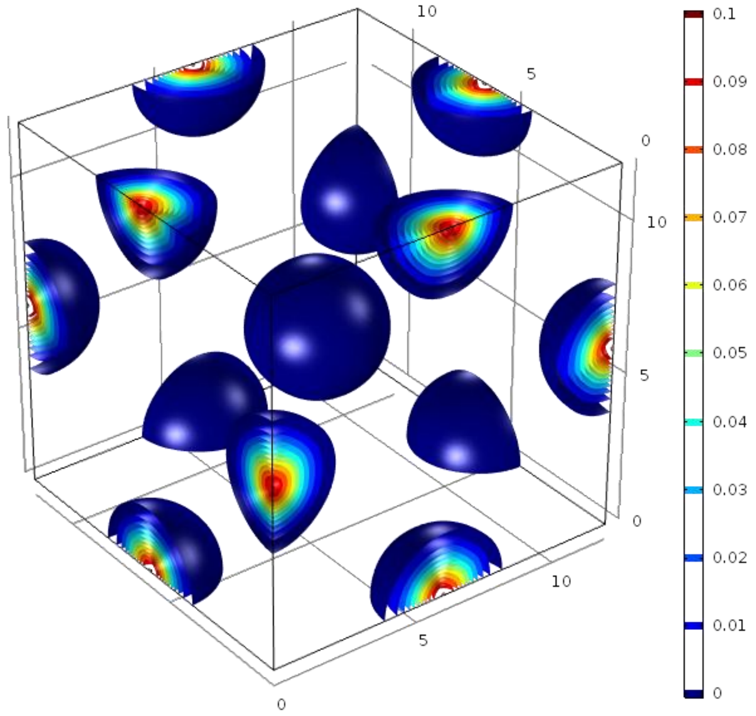
Face-centered cubic lattice

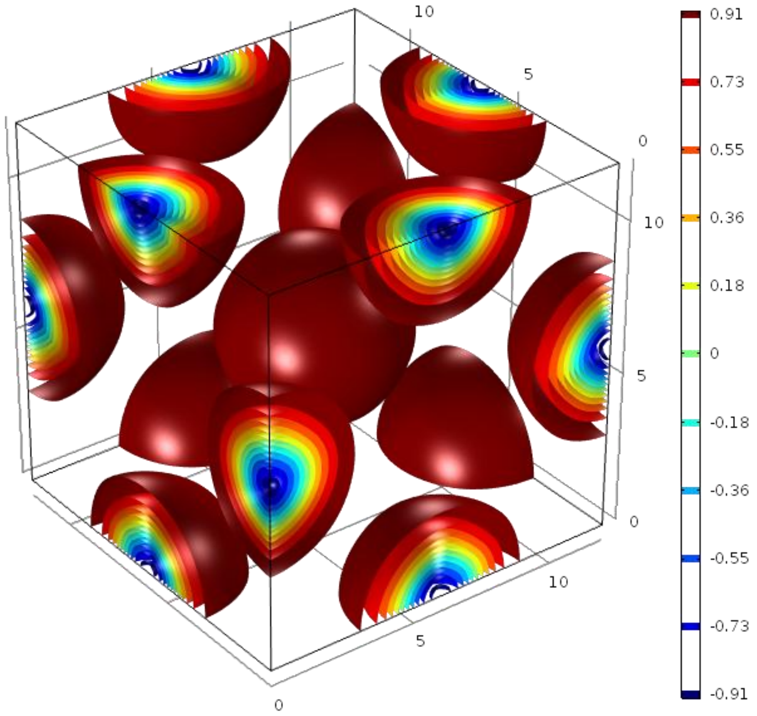
$$(x, y, z) \to (-x, y, z) \Rightarrow (\sigma, \pi_1, \pi_2, \pi_3) \to (\sigma, -\pi_1, \pi_2, \pi_3)$$

$$(x, y, z) \to (z, x, y) \Rightarrow (\sigma, \pi_1, \pi_2, \pi_3) \to (\sigma, \pi_3, \pi_1, \pi_2)$$

$$(x, y, z) \to (x, z, -y) \Rightarrow (\sigma, \pi_1, \pi_2, \pi_3) \to (\sigma, \pi_1, \pi_3, -\pi_2)$$

$$(x, y, z) \to (x + L, y + L, z) \Rightarrow (\sigma, \pi_1, \pi_2, \pi_3) \to (\sigma, -\pi_1, -\pi_2, \pi_3)$$





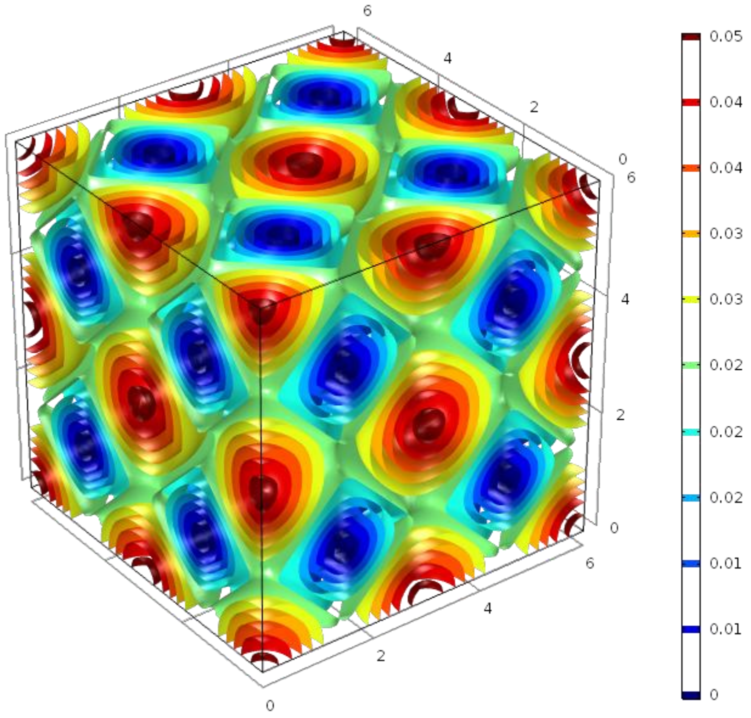
Simple cubic lattice

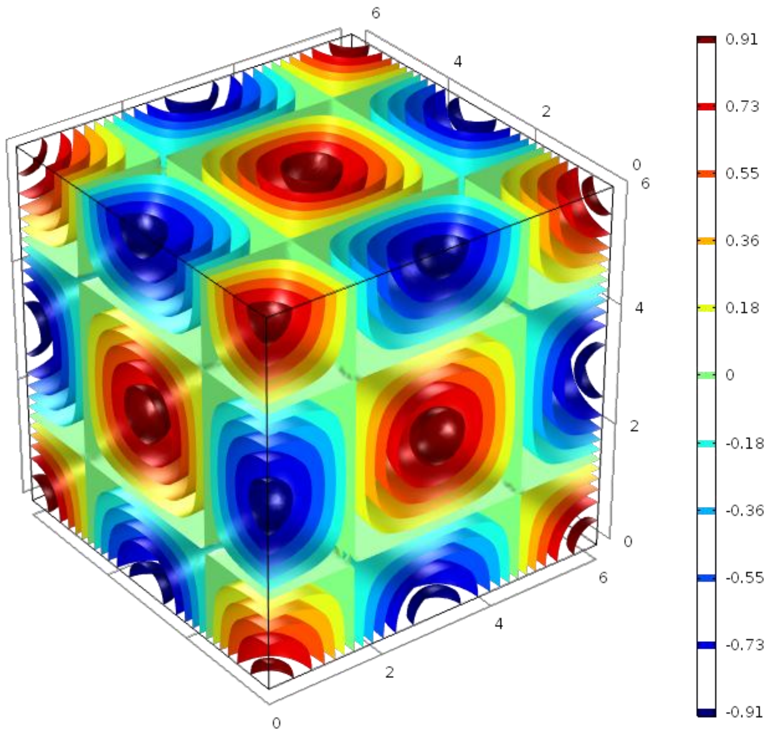
$$(x, y, z) \to (-x, y, z) \Rightarrow (\sigma, \pi_1, \pi_2, \pi_3) \to (\sigma, -\pi_1, \pi_2, \pi_3)$$

$$(x, y, z) \to (z, x, y) \Rightarrow (\sigma, \pi_1, \pi_2, \pi_3) \to (\sigma, \pi_3, \pi_1, \pi_2)$$

$$(x, y, z) \to (x, z, -y) \Rightarrow (\sigma, \pi_1, \pi_2, \pi_3) \to (\sigma, \pi_1, \pi_3, -\pi_2)$$

$$(x, y, z) \to (x + L, y, z) \Rightarrow (\sigma, \pi_1, \pi_2, \pi_3) \to (\sigma, -\pi_1, \pi_2, \pi_3)$$





Numerical method

$$\bar{\sigma} = \sum_{a,b,c} \beta_{abc} \cos\left(\frac{a\pi x}{L}\right) \cos\left(\frac{b\pi y}{L}\right) \cos\left(\frac{c\pi z}{L}\right)$$

$$\beta_{abc} = \beta_{bca} = \beta_{cab}$$

$$\overline{\pi_1} = \sum_{a,b,c} \alpha_{abc} \sin\left(\frac{a\pi x}{L}\right) \cos\left(\frac{b\pi y}{L}\right) \cos\left(\frac{c\pi z}{L}\right)$$

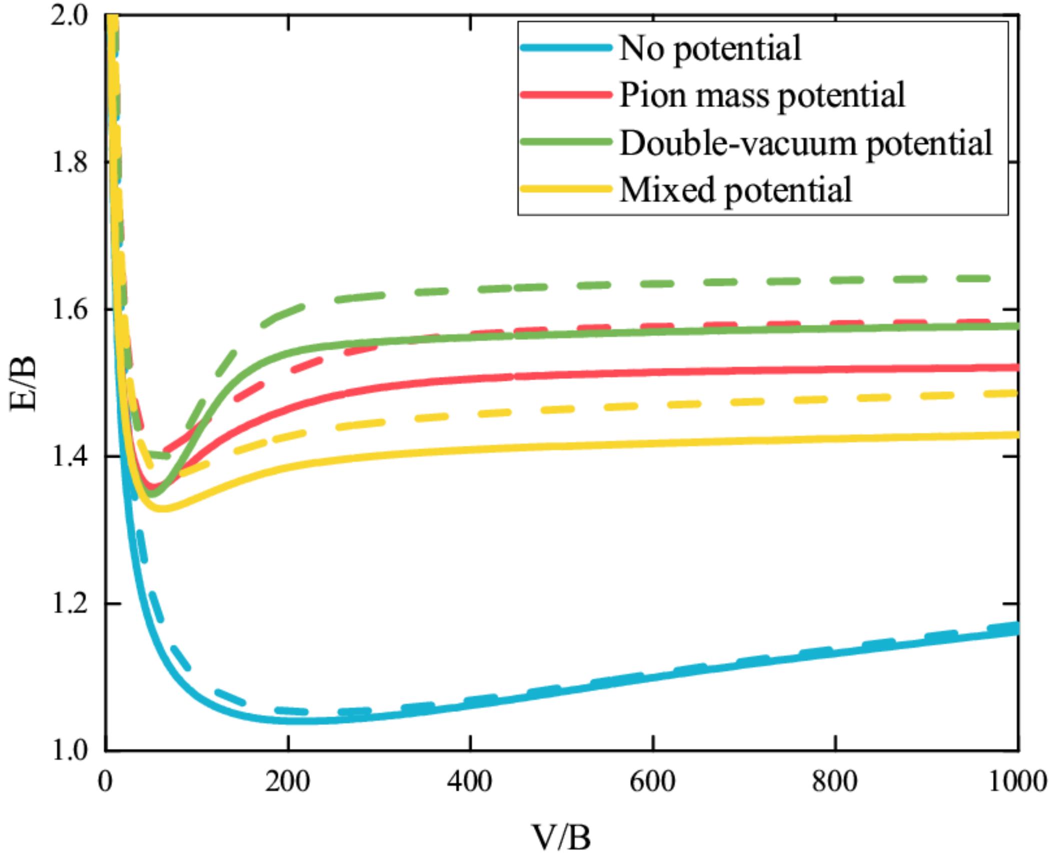
$$\overline{\pi_2} = \sum_{a,b,c} \alpha_{abc} \sin\left(\frac{a\pi z}{L}\right) \cos\left(\frac{b\pi x}{L}\right) \cos\left(\frac{c\pi y}{L}\right)$$

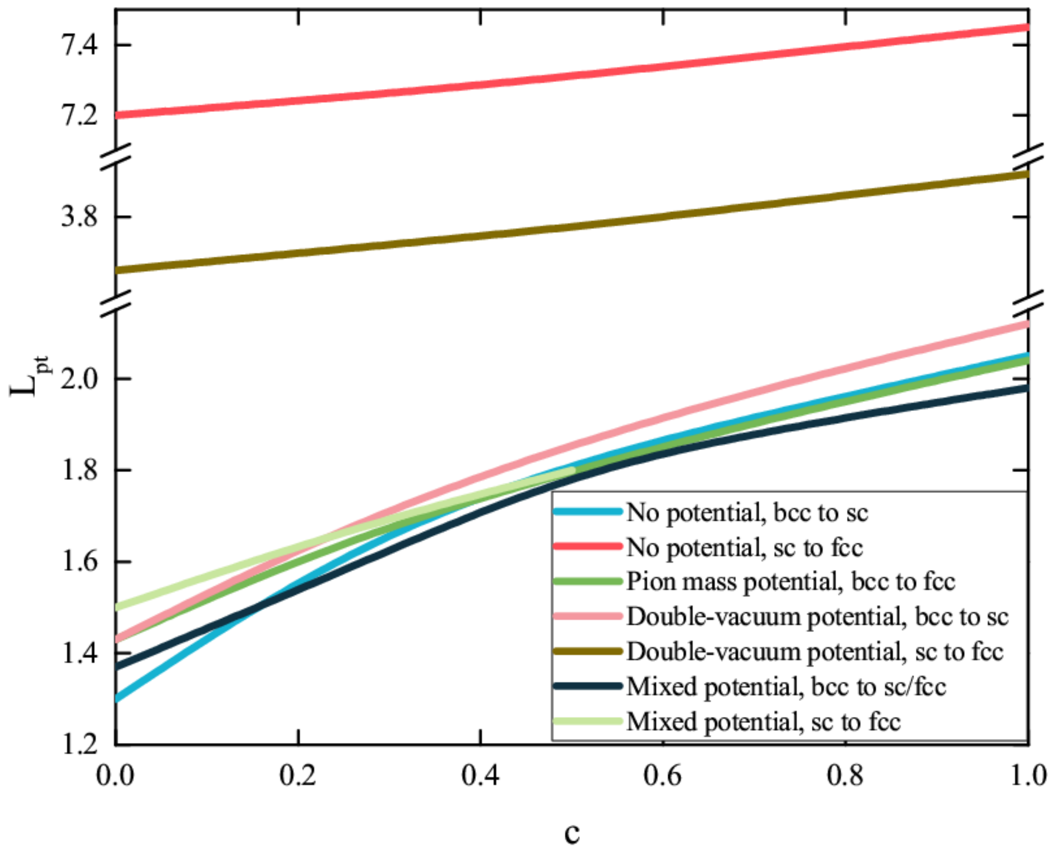
$$\overline{\pi_3} = \sum_{a,b,c} \alpha_{abc} \sin\left(\frac{a\pi y}{L}\right) \cos\left(\frac{b\pi z}{L}\right) \cos\left(\frac{c\pi x}{L}\right)$$

SNOPT →

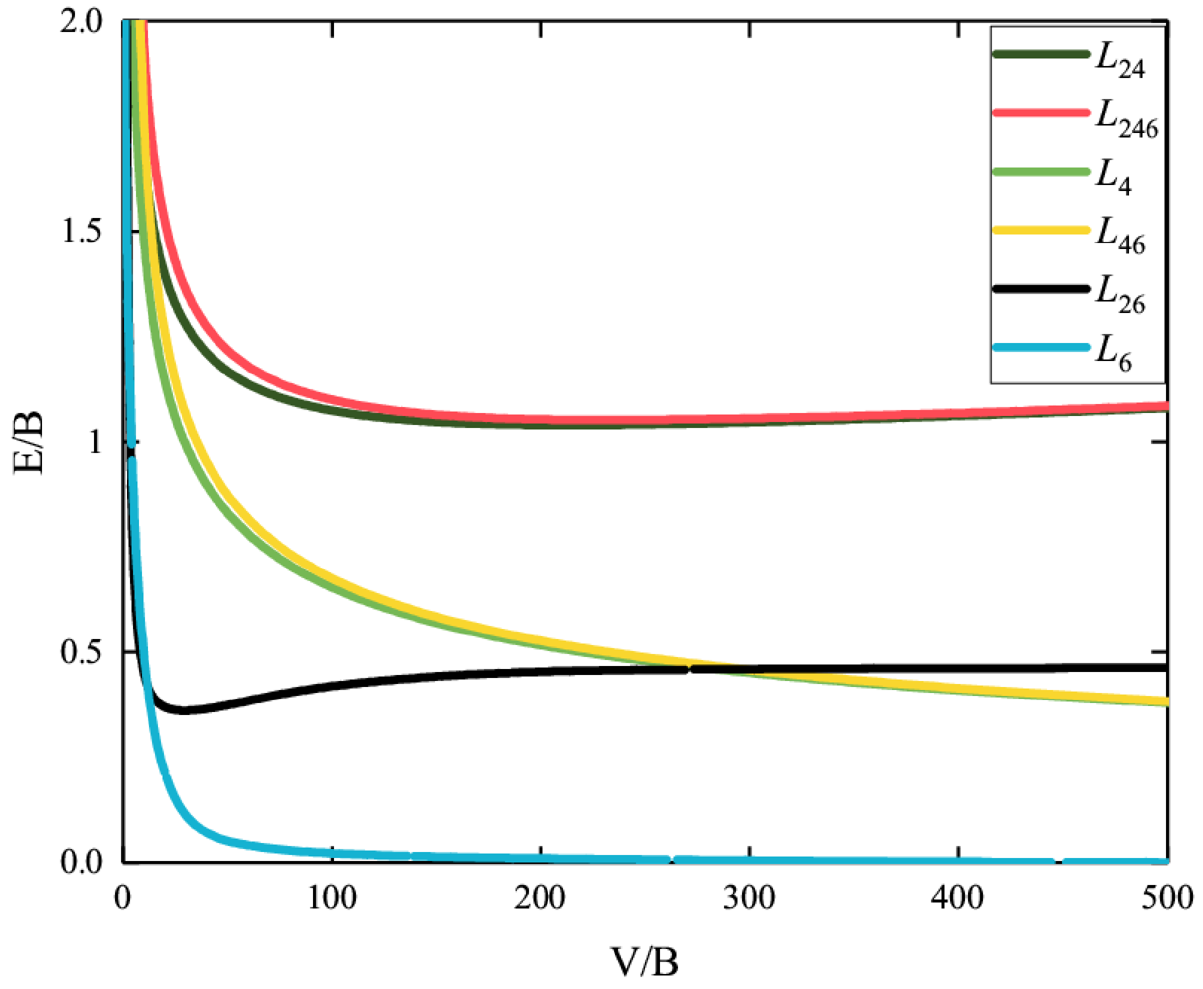
$$\boldsymbol{n} = \frac{\overline{\boldsymbol{n}}}{\|\overline{\boldsymbol{n}}\|}$$

Numerical results: full model

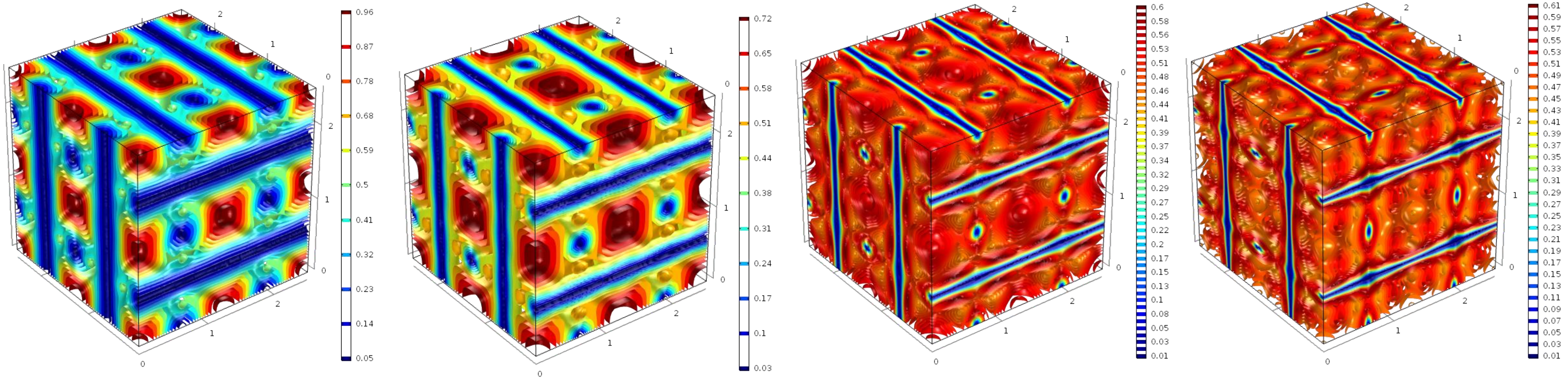




Numerical results: limiting cases

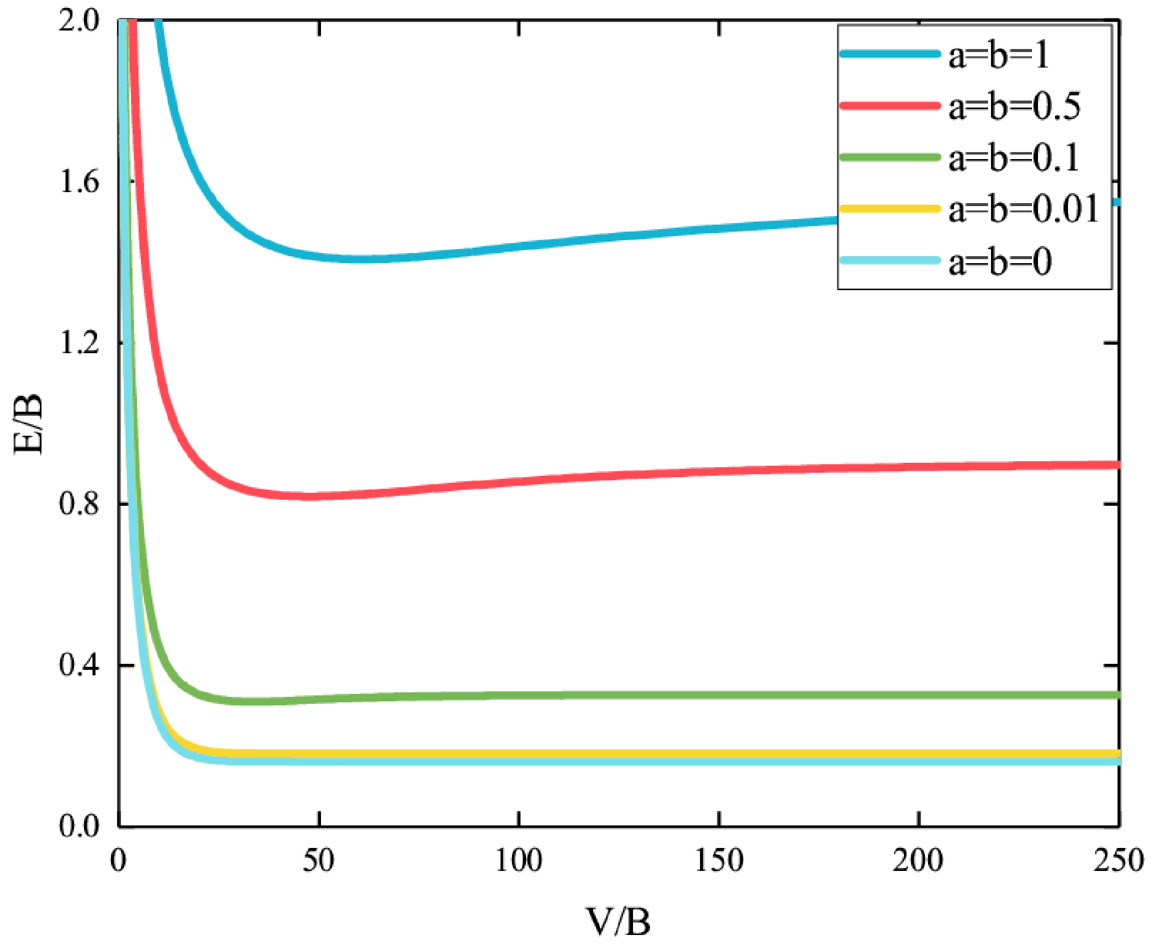


Numerical results: limiting cases



$\mathcal{L}_4$ $\mathcal{L}_{26}$ $\mathcal{L}_{46}$ $\mathcal{L}_6$

Numerical results: transition to self-dual limit



Conclusions

- Generalized Skyrme model allows four different phases for its crystal-like solutions
- Inclusion of sixth order term always increase energy of crystal and shift position of minimum of energy of crystal to lower values of density
- Phase structure of crystal strongly depends on choice of potential and values of coupling constants
- Crystal-like solutions exist in $\mathcal{L}_4$, $\mathcal{L}_6$ и $\mathcal{L}_{46}$ submodels that do not possess free solitonic solutions
- Energy of crystal in $\mathcal{L}_4$, $\mathcal{L}_6$ и $\mathcal{L}_{46}$ submodels does not possess global minimum
- Indications for transistion from quasiliquid low-density phase to high-density highly symmetric phase are observed when approaching self-dual limit

Thank you for attention