Stationary non-topological solitons in 3+1 dimension field theory

VICTOR LOIKO

DEPARTMENT OF THEORETICAL PHYSICS AND ASTROPHYSICS BSU

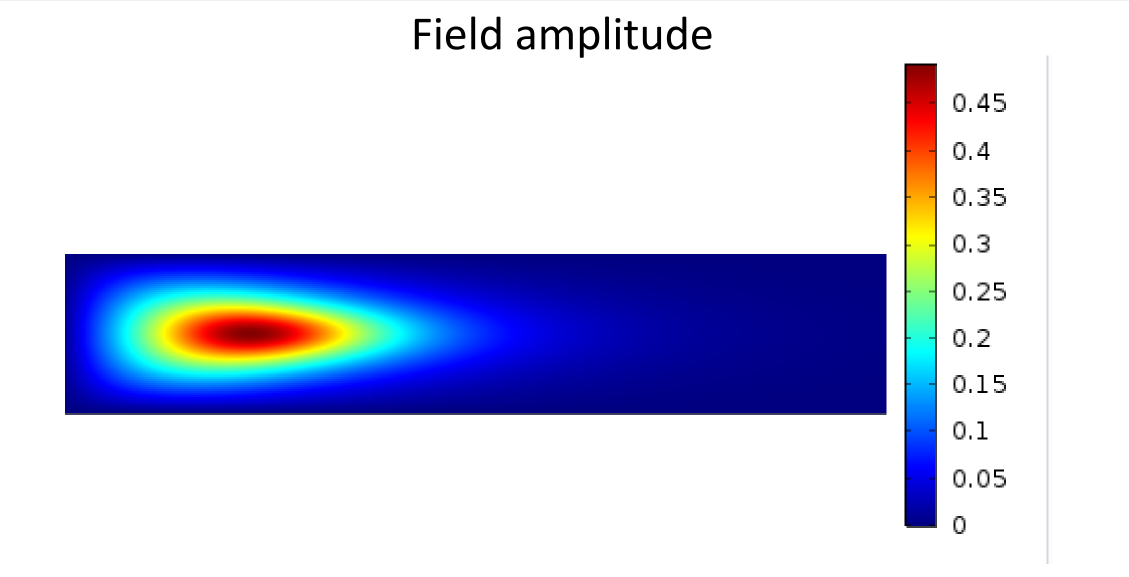
MINSK

PROJECT SUPERVISOR: PROF. YA. SHNIR

Solitons

Solitons are solutions of nonlinear partial differential equation which represent a solitary travelling wave, which:

- ➢Doesn't obey the superposition principle
- ➢Possessed finite stable shape in space
- ➢Nondispersive



Q-balls – non-topological solitons with time dependent field, carrying conserving Noether charge associated with symmetry

Ansatz

R. Friedberg. T.D. Lee and A. Sirlin. Phys. Rev. D 13 (1976)

$$L = -\partial^{\mu}\phi^{+}\partial_{\mu}\phi - \frac{1}{2}\partial^{\mu}\chi\partial_{\mu}\chi - f^{2}\chi^{2}\phi^{+}\phi - V(\chi)$$

$$\phi \to e^{i\alpha}\phi$$

$$\chi \to -\chi$$

$$Q = i\int_{-\infty}^{+\infty} dx(\phi\partial_t\phi^{*} - \phi^{*}\partial_t\phi)$$

$$E = T^{00} = \frac{\partial L}{\partial(\partial_0\phi)}\partial^{0}\phi - g^{00}L$$

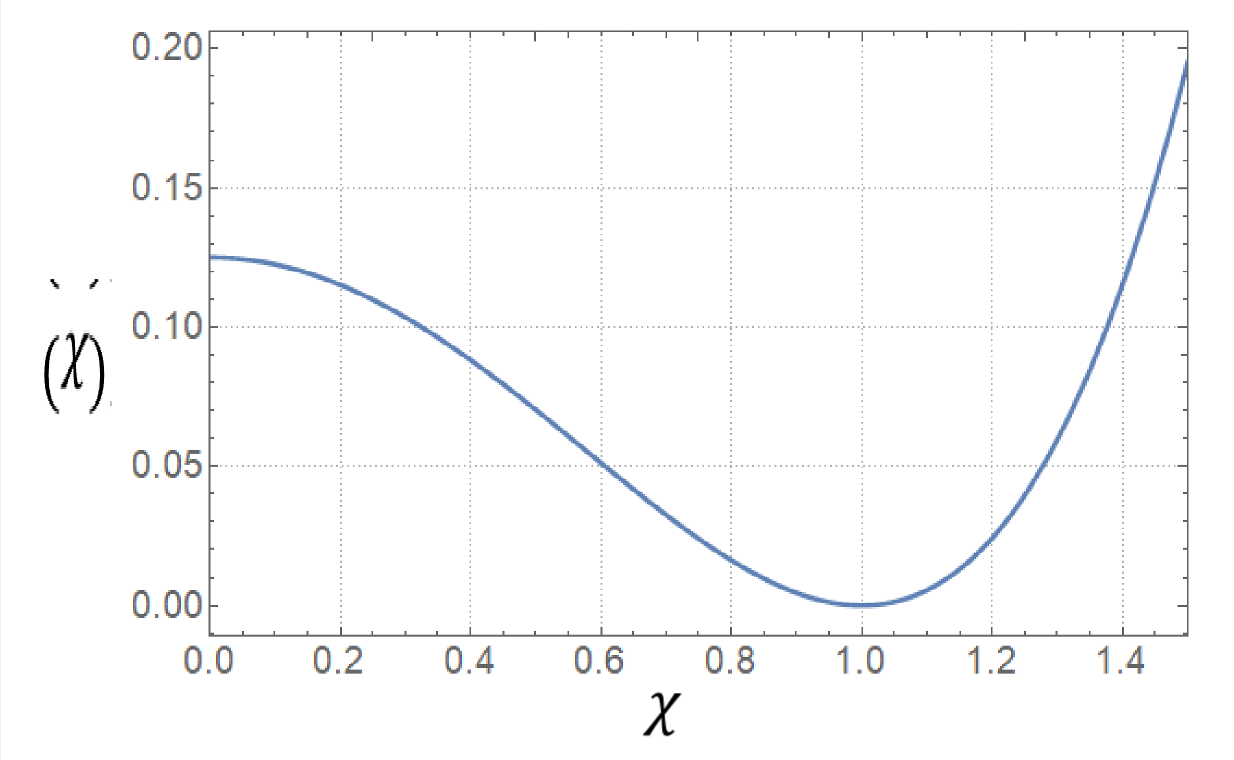
ϕ – complex field
χ – scalar field
$V(\chi)$ -- potential of scalar field
Q – Noether charge

Potential

$$V(\chi) = \frac{1}{8} g^2 (\chi^2 - {\chi_{vac}}^2)^2$$

$$V(\infty) = \infty$$

$$V(\chi_{vac}) = 0$$



Field equation

$$\frac{\partial L}{\partial \psi} = \partial^{\mu} \frac{\partial L}{\partial(\partial_{\mu}\psi)}$$

$$\partial^{\mu}\partial_{\mu}\chi = 2f^{2}\chi\phi^{+}\phi + \frac{1}{2}g^{2}(\chi^{2} - \chi_{vac}{}^{2})\chi$$

$$\partial^{\mu}\partial_{\mu}\phi^{+} = f^{2}\chi^{2}\phi^{+}$$

$$\chi(r) = \chi_{vac}A(\rho)$$

$$\phi(r,t) = \frac{1}{\sqrt{2}}\chi_{vac}B(\rho)e^{-i\omega t}$$

$$\rho = \mu r \qquad \mu = g\chi_{vac}$$

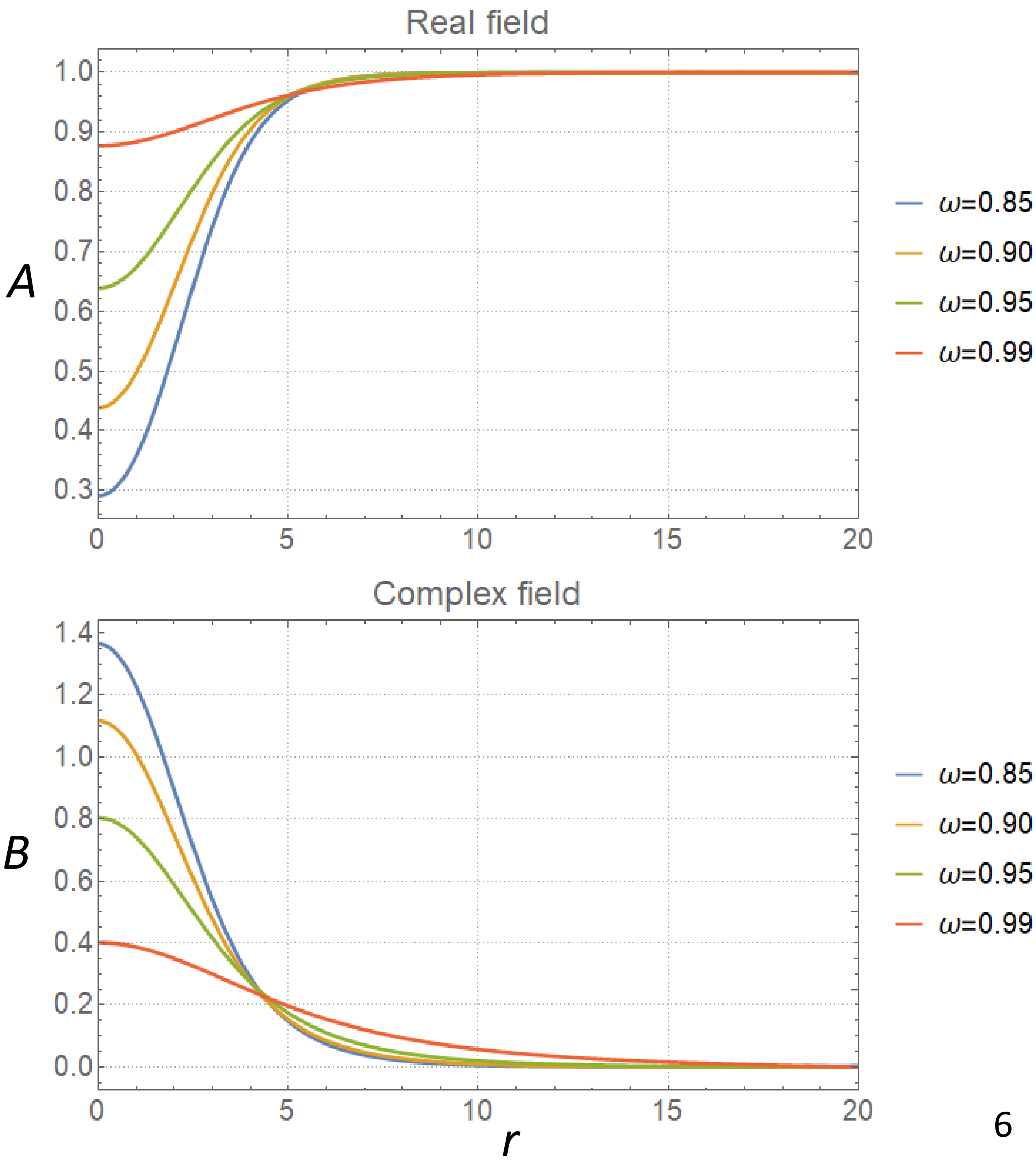
Solutions

$$\Delta A - k^2 B^2 A - \frac{1}{2}\left(A^2 - 1\right) A = 0$$
$$\Delta B - k^2 A^2 B + \upsilon^2 B = 0$$

$$\rho \to 0: \qquad \frac{dA}{d\rho} = \frac{dB}{d\rho} = 0$$

$$\rho \to \infty: \qquad A = 1; B = 0$$

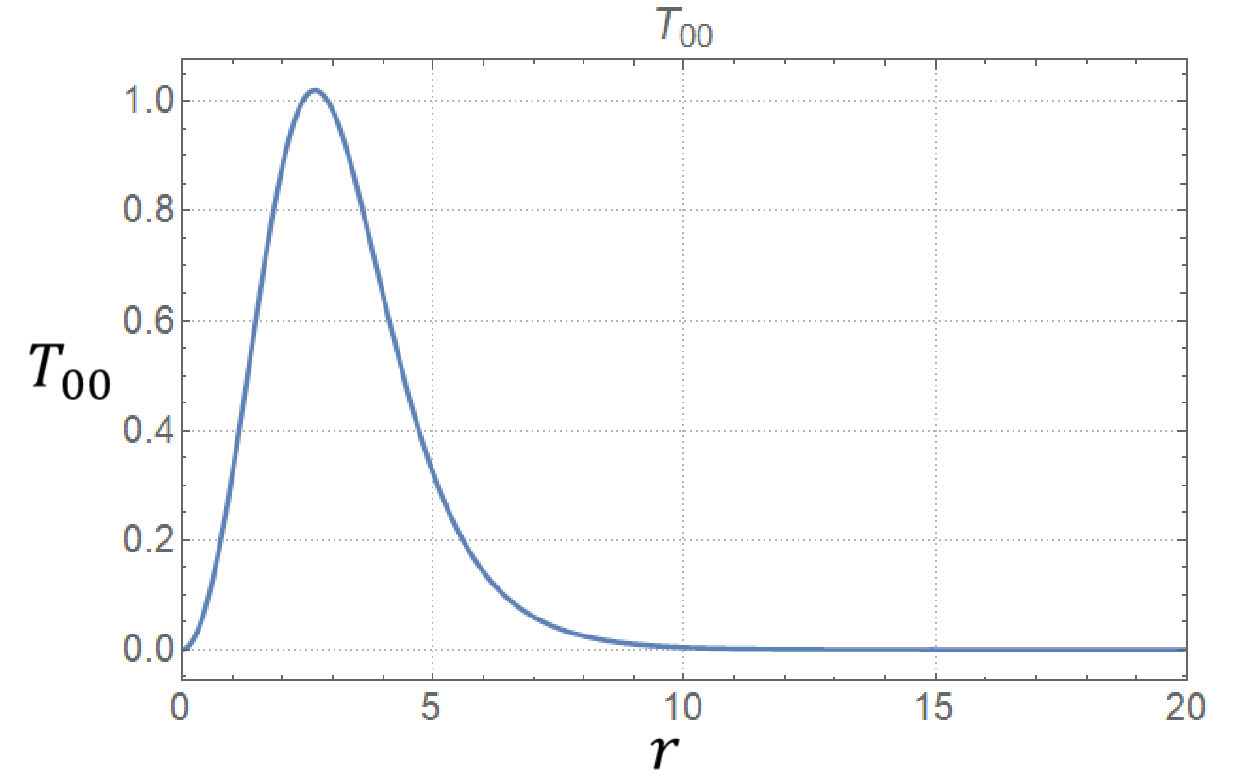
$\upsilon = \omega/\mu$ $\qquad$ $k = m/\mu$



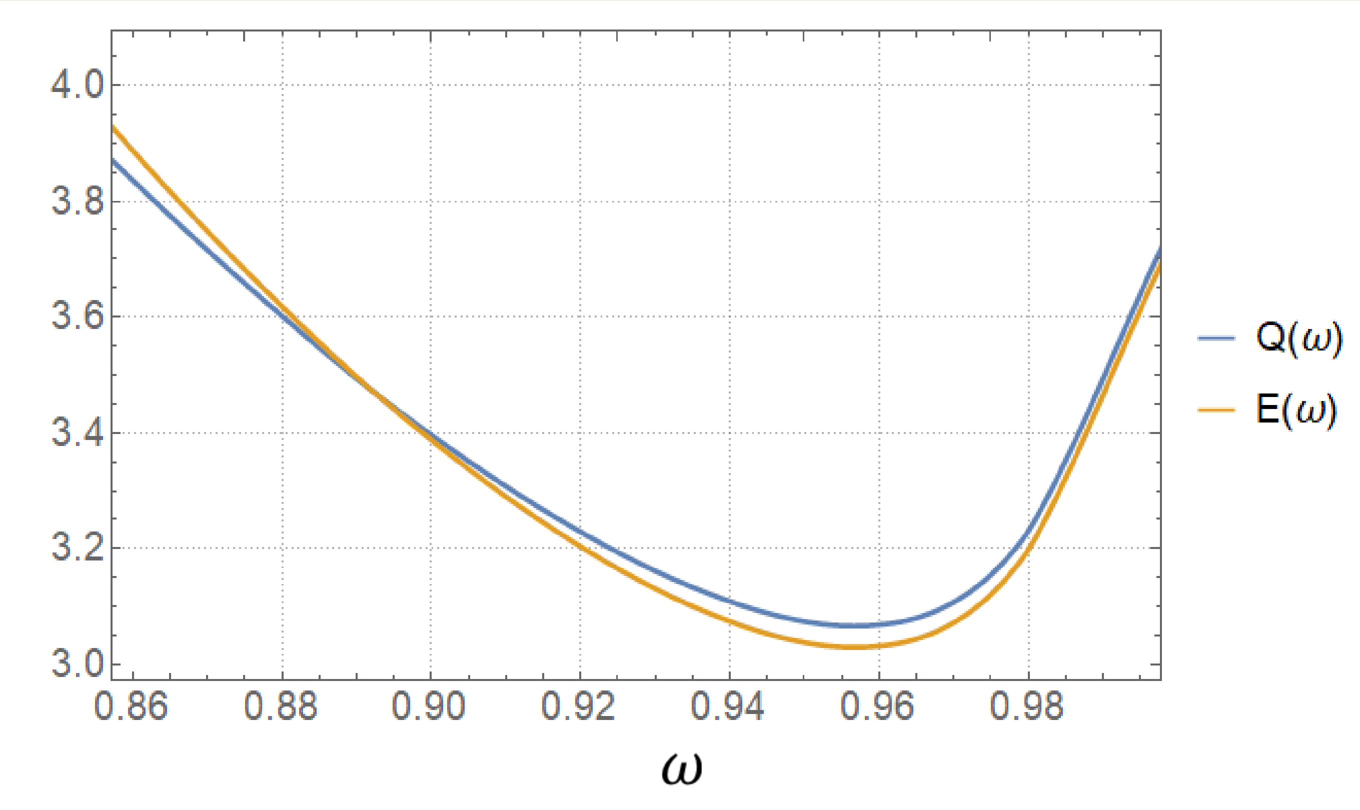
Energy and charge

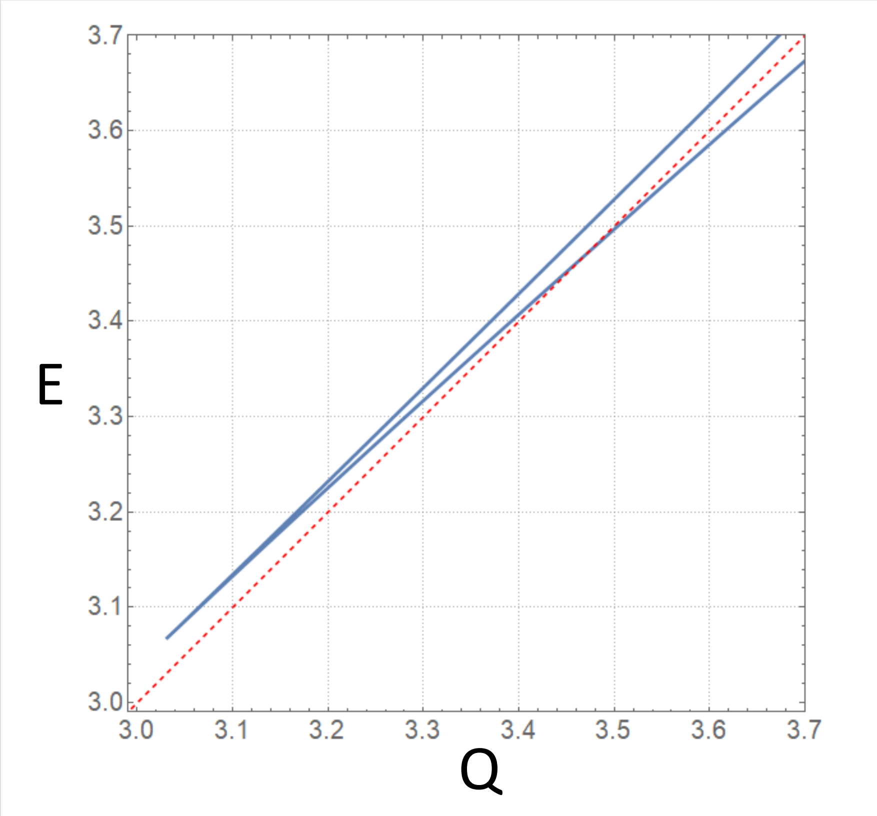
$$Q = \frac{\upsilon}{2}\int B^2\rho^2 d\rho = \frac{\upsilon}{2}N$$

$$E = \frac{1}{2k}\int\left(\frac{1}{2}(\nabla A)^2 + \frac{1}{2}(\nabla B)^2 + \frac{1}{2}(\upsilon^2 + k^2A^2)B^2 + \frac{1}{2}(A^2-1)^2\right)\rho^2 d\rho$$



Energy and charge values





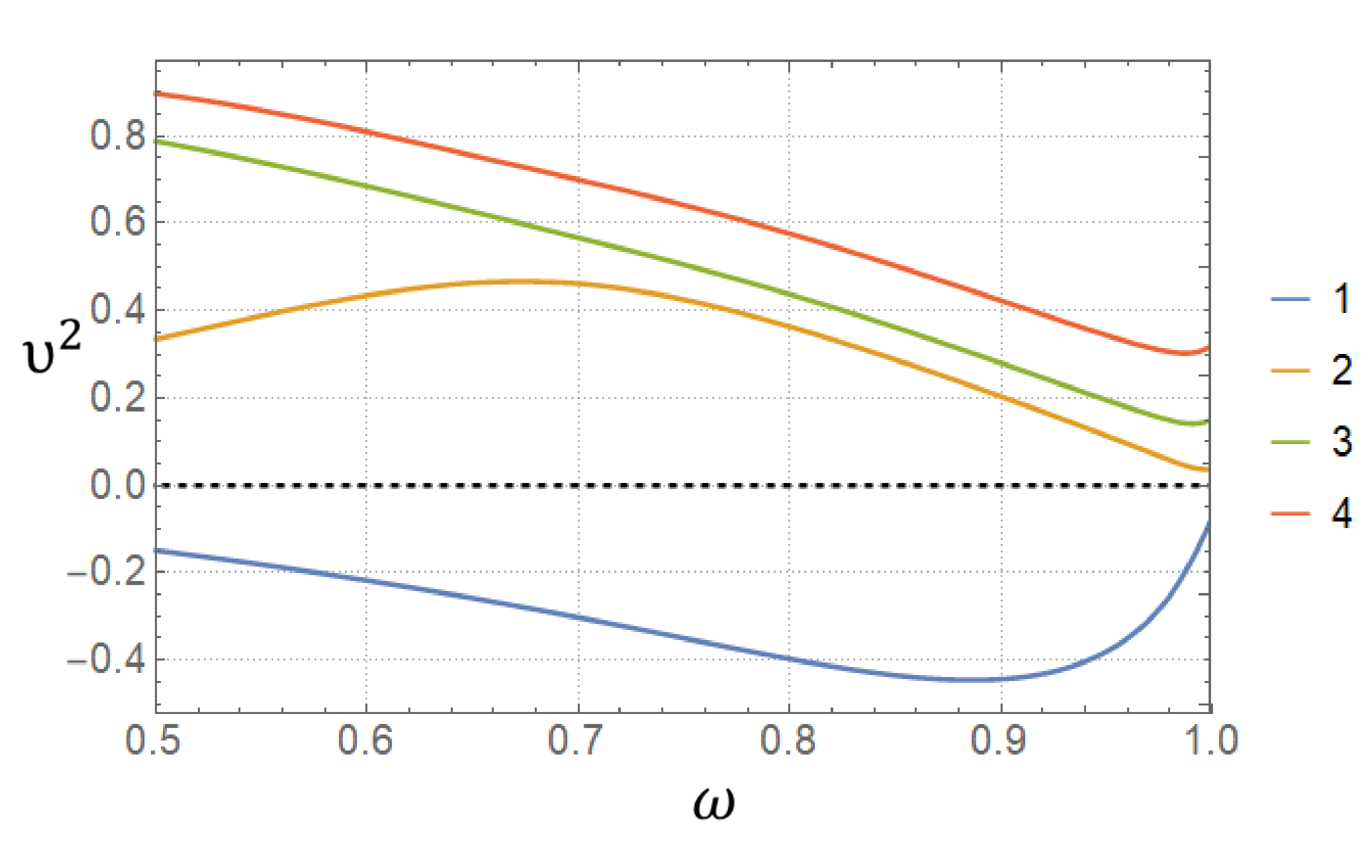
Stability analysis

$$\chi(r,t) = A(r,t)$$
$$\phi(r,t) = B(r,t)e^{-i\omega t}$$

$$A(r,t) = A_0(r) + f\, A_1(r,t) + O(f)^2$$
$$B(r,t) = B_0(r) + f\, B_1(r,t) + O(f)^2$$

$$A_1(r,t) = A_2(r)e^{i\upsilon t}$$
$$B_1(r,t) = B_2(r)e^{i\upsilon t}$$

$$A_2'' + F_1(r)A_2' + F_2(r)A_2 = \upsilon^2 A_2$$
$$B_2'' + G_1(r)B_2' + G_2(r)B_2 = \upsilon^2 B_2$$

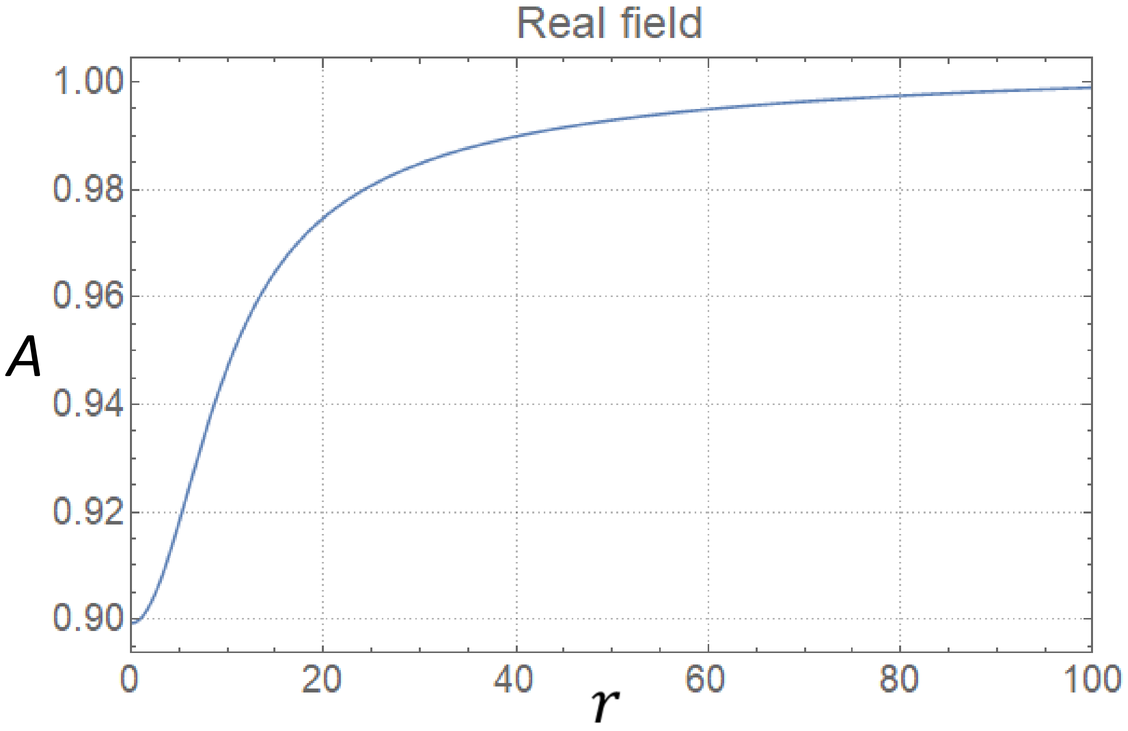


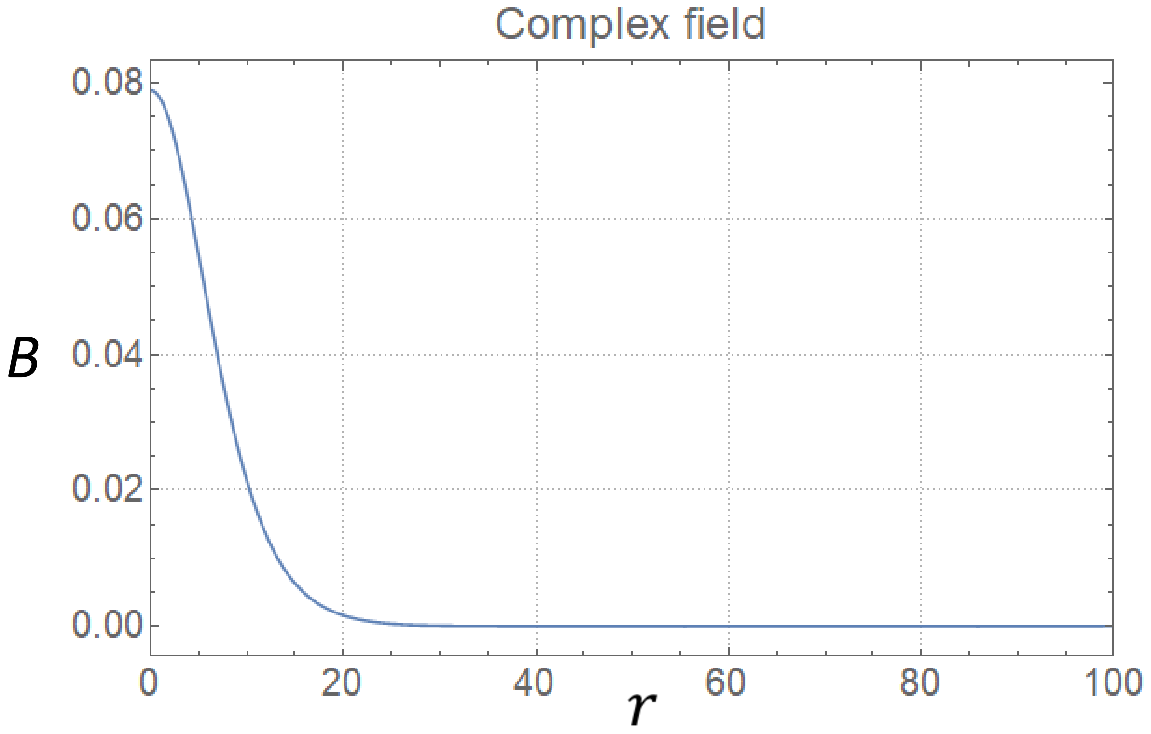
Massless limit

A. Levin, V. Rubakov arXiv:1010.0030v1 (2010)

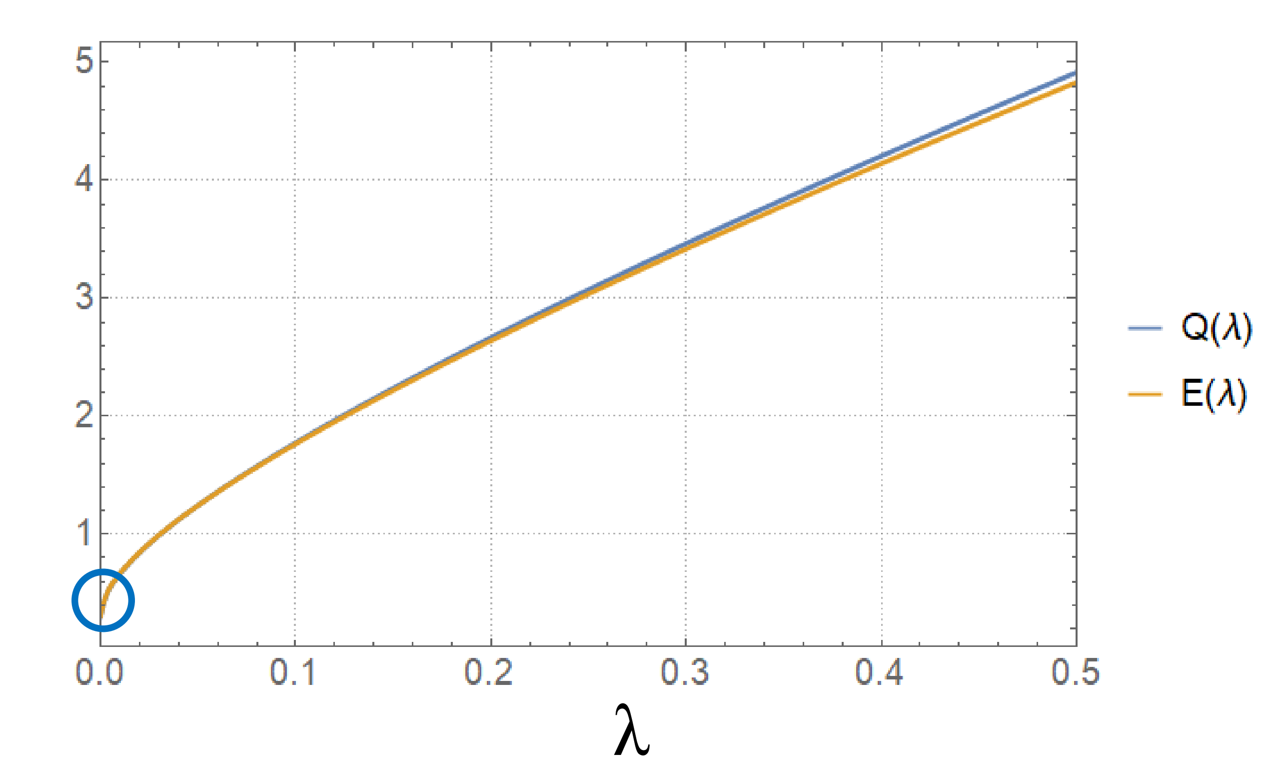
$$V(A) = 2\lambda(A^2 - 1)^2$$

$$\lambda \to 0$$





Massless limit



Spinning FLS Q-ball

$$\chi(r,\theta) = A(r,\theta)$$
$$\phi(r,\theta,\varphi,t) = B(r,\theta)e^{-i\omega t + iN\varphi}$$

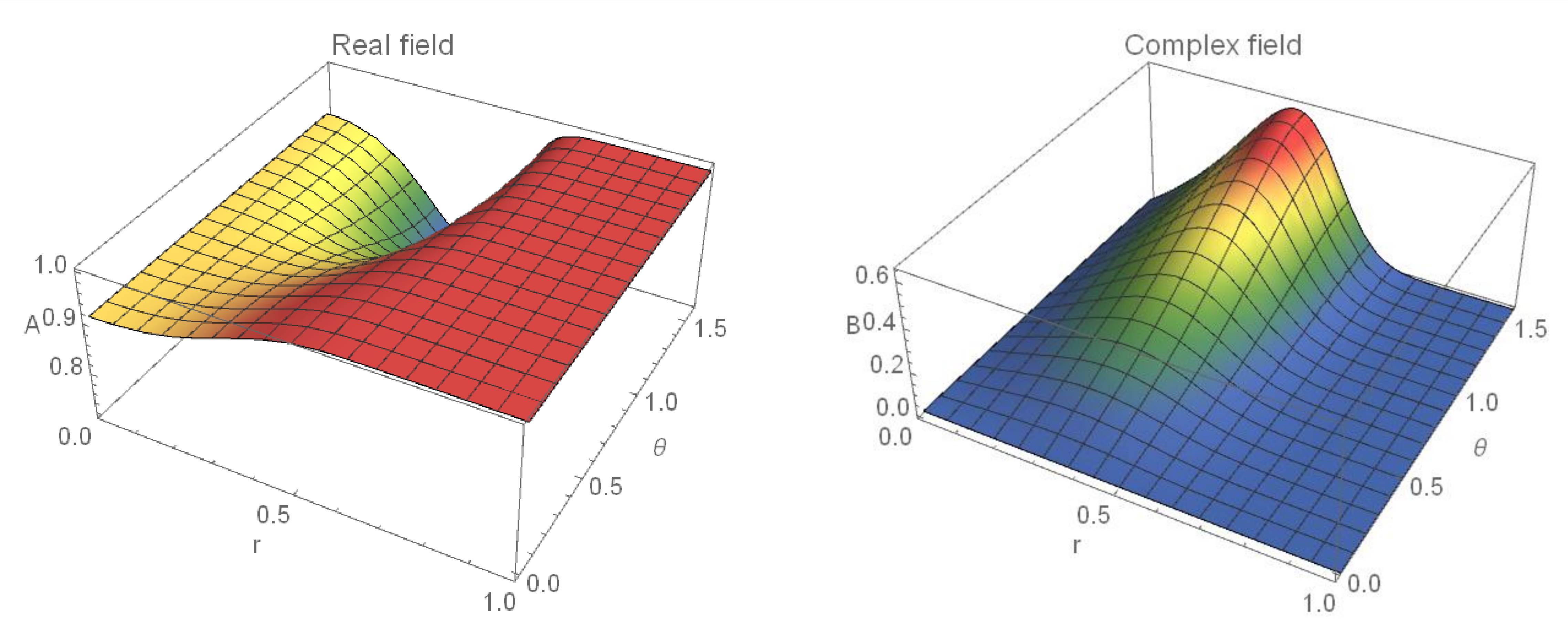
$$\frac{\partial^2 A}{\partial r^2} + \frac{2}{r}\frac{\partial A}{\partial r} + \frac{\cos\theta}{r^2 \sin\theta}\frac{\partial A}{\partial \theta} + \frac{1}{r^2}\frac{\partial^2 A}{\partial \theta^2} - k^2 B^2 A - 2\lambda(A^2 - 1)\,A = 0$$

$$\frac{\partial^2 B}{\partial r^2} + \frac{2}{r}\frac{\partial B}{\partial r} + \frac{\cos\theta}{r^2 \sin\theta}\frac{\partial B}{\partial \theta} + \frac{1}{r^2}\frac{\partial^2 B}{\partial \theta^2} - \frac{n^2}{r^2 \sin^2(\theta)}B - k^2 A^2 B + \upsilon^2 B = 0$$

$$\frac{\partial A}{\partial r}(0,\theta) = 0; \quad \frac{\partial A}{\partial \theta}(r,0) = 0$$
$$A(\infty,\theta) = 1; \quad \frac{\partial A}{\partial \theta}(r,\pi/2) = 0$$

$$B(0,\theta) = 0; \quad B(r,0) = 0$$
$$B(\infty,\theta) = 0; \quad \frac{\partial A}{\partial \theta}(r,\pi/2) = 0$$

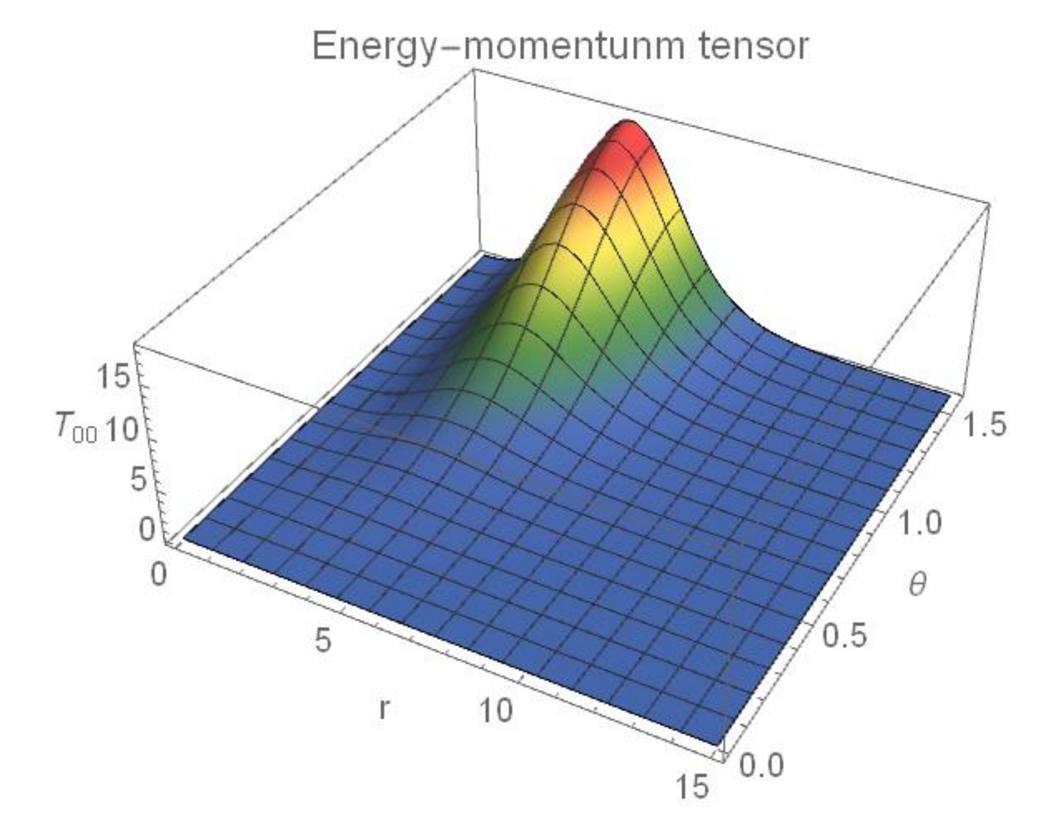
Spinning FLS Q-ball



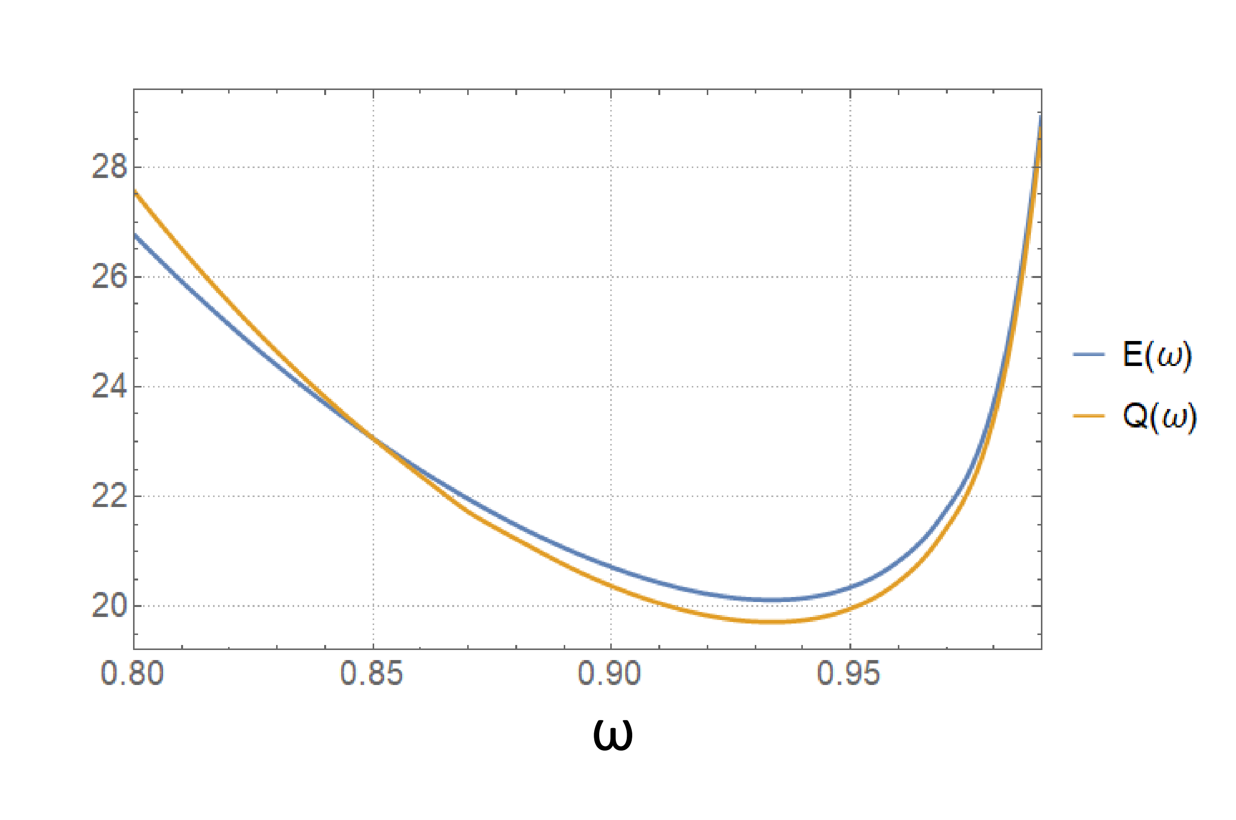
Energy and charge

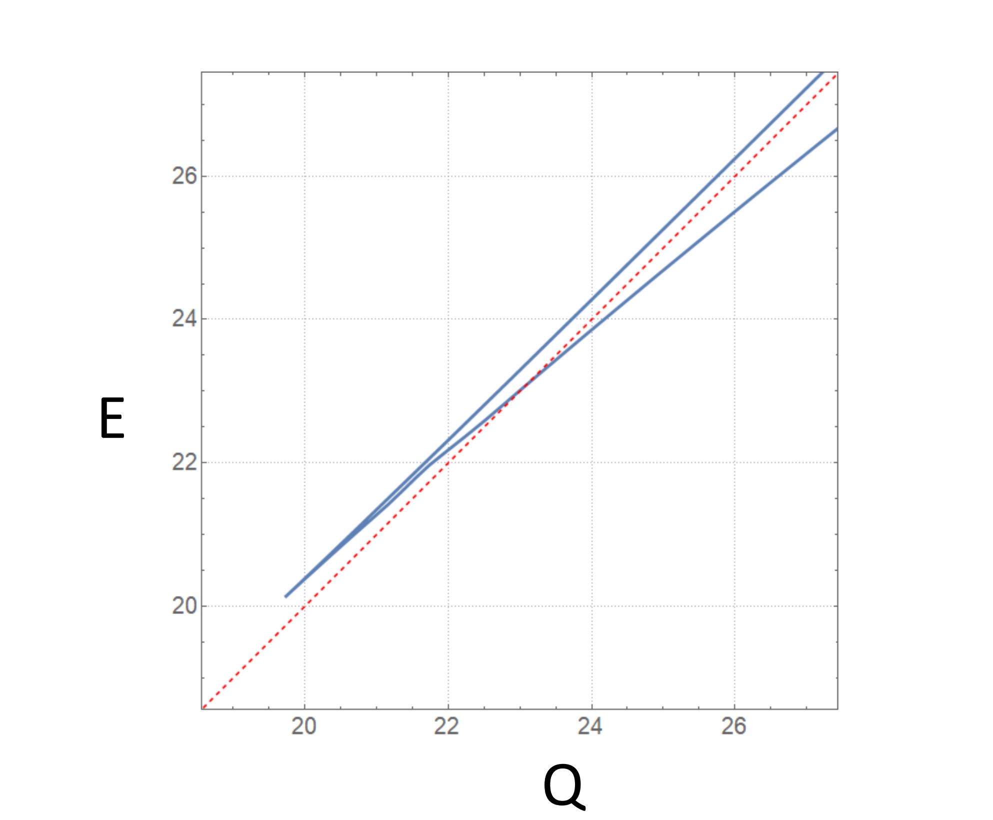
$$Q = 8\pi\omega \int_0^{\pi/2} d\theta \int_0^{\infty} B^2 r^2 \sin\theta \, dr$$

$$E = 4\pi \int_0^{\pi/2} d\theta \int_0^{\infty} \left(\omega^2 f^2 + (\partial_r A)^2 + (\partial_r B)^2 + \frac{1}{r^2}(\partial_\theta A)^2 + \frac{1}{r^2}(\partial_\theta B)^2 + kA^2B^2 + \frac{n^2B^2}{r^2\sin^2(\theta)} + V\right) r^2 \sin\theta \, dr$$



Energy and charge values





Conclusions

- Solitons of Q-ball type appear in model with conserving charge and corresponding symmetry doesn’t break spontaneously
- Existence and stability of soliton configuration connected with minimum energy state across all configurations with fixed charge value.
- Energy vs charge dependency has two branches in dependence on parameter and one of them stable and another is unstable
- In massless limit Q-balls stabilized by the gradient energy of real field rather than potential energy
- Massless limit needs further investigation to verify does compacton has a place.

Thanks for your attention!