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Quantum field theory methods in non-linear stochastic dynamics

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Turbulent motion

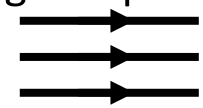
Reynolds number - measure of the ratio of inertia and viscosity forces acting in liquid.

$$\mathrm{Re} = \frac{UL}{\nu}$$

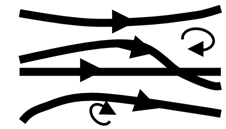
L - characteristic size scale

U - characteristic velocity scale

$Re < Re_{kr}$ → laminar flow



$Re > Re_{kr}$ → turbulence



Prandtl number – indicates domination of thermal/molecular or momentum diffusivity

$$\mathrm{Pr} = \frac{\nu}{\alpha}$$

ν – kinematic viscosity

α – thermal/molecular diffusivity

small → heat/molecules diffuse faster than velocity

turbulence → turbulent Prandtl number

Our case

- ✓ Fully developed turbulence
- ✓ Effective values of diffusion coefficients
- ✓ $Re \to \infty$
- ✓ Pr – universal constant

Fluid flow

- Stochastic **Navier – Stokes (NS)** equation

$$\partial_t v_i = \nu_0 \Delta v_i - (v_j \partial_j) v_i + \partial_i P + f_i$$

Incompressible: $\partial_i v_i = 0$

ν_0 - kinematic viscosity;
P – pressure;
v_i - velocity.

Transverse external random force

✓ Gaussian distribution; ✓ Zero mean value; ✓ Correlation function:

$$\langle f_i(t,x) f_j(t',x') \rangle = \delta(t-t') \frac{1}{(2\pi)^d} \int d\vec{k} P_{ij}(\vec{k}) e^{-i\vec{k}(\vec{x}-\vec{x}\prime)} d_f(k)$$

Transverse projection operator:

$$P_{ij}(k) = \delta_{ij} - \frac{k_i k_j}{k^2}$$

"Pumping" function:

$$d_f = g_0 \nu_0^3 k^{4-d-2\varepsilon}$$

Physical value $\varepsilon = 2$.

Passive scalar field

- ✓ Physical meanings : temperature, concentration of admixture particles etc.;
- ✓ Passive: no influence on solution of NS equation.

➢ **Turbulent mixing**:

$$\partial_t \varphi = \underbrace{\nu_0 \mu_0} \Delta \varphi - (v \cdot \nabla) \varphi$$

ν_0 - kinematic viscosity;
μ_0 – inverse Prandtl number;
φ - scalar field.

coefficient of molecular diffusion

coefficient of thermal conductivity

Equivalent QFT model

Model with doubled number of fields: $v, \varphi \rightarrow \Phi = \{v, \varphi, v', \varphi'\}$

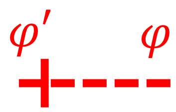
Of the same nature as v, φ

Corresponding action:

$$S(\Phi) = \frac{1}{2} v' D_f v' + v'[-\partial_t v + \nu_0 \Delta v - (v \cdot \nabla) v] + \varphi'[-\partial_t \varphi + \nu_0 \mu_0 \Delta \varphi - (v \cdot \nabla) \varphi]$$

Propagators:

$$\Delta^{\varphi'\varphi}(k) = \frac{1}{i\omega_k + \nu_0 \mu_0 k^2}$$

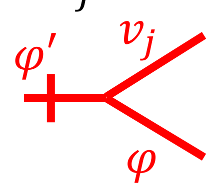


$$\Delta_{ij}^{v'v}(k) = \frac{P_{ij}(k)}{i\omega_k + \nu_0 k^2}$$

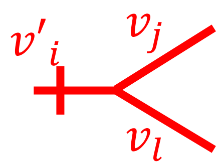
$$\Delta_{ij}^{vv}(k) = \frac{g_0 \nu_0^3 P_{ij}(k)}{(i\omega_k + \nu_0 k^2)(-i\omega_k + \nu_0 k^2)}$$

Vertices:

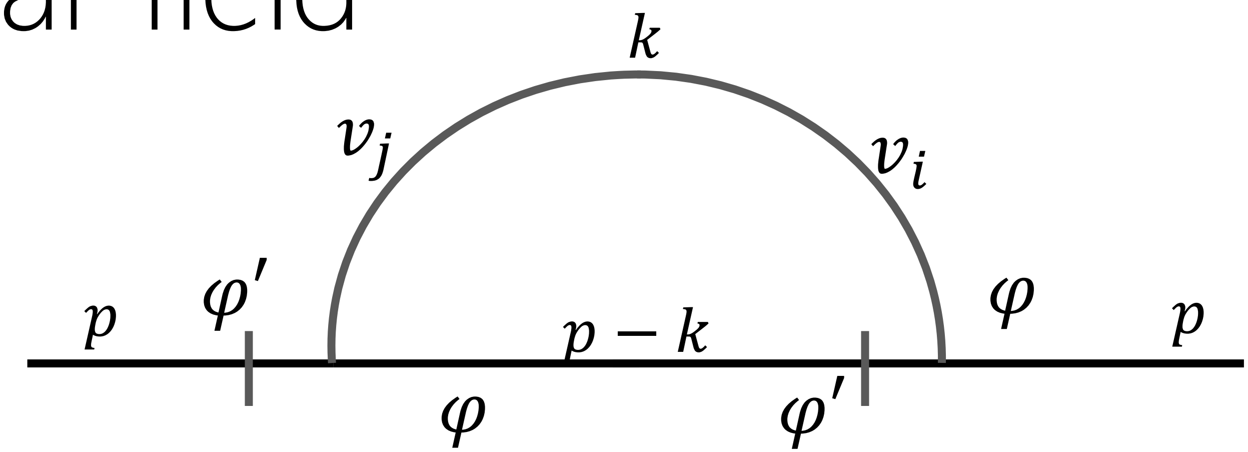
$$V_j(k) = i k_j$$



$$V_{ijl}(k) = i(\delta_{ij} k_l + \delta_{il} k_j)$$



Scalar field



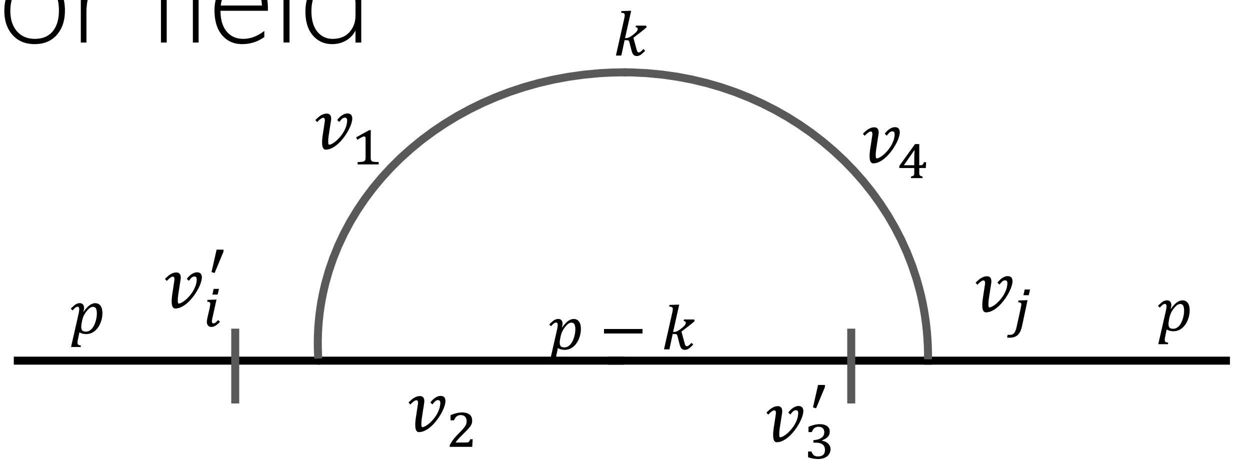
$$\sum^{\varphi'\varphi}(k,\omega) = \int d\omega_k \int d^d k V_j(p)\, V_i(p-k)\Delta_{ij}^{vv}(k)\Delta^{\varphi\varphi'}(p-k)\Big)$$

$$\sum^{\varphi'\varphi} = -\frac{g}{\varepsilon}\frac{(d-1)}{u(u+1)}\frac{1}{4d}\frac{S_d}{(2\pi)^d}$$

$$S_d = \frac{2\pi^{d/2}}{\Gamma\left(\frac{d}{2}\right)}$$

surface of d-dimensional sphere

Vector field



$$\sum_{ij}^{v'v}(k,\omega) = \int d\omega_k \int d^d k V_{i12}(p) V_{34j}(p-k) \Delta_{14}^{vv}(k) \Delta_{23}^{vv'}(p-k) \Big)$$

$$\sum_{ij}^{v'v} = -\frac{g}{\varepsilon}\frac{(d-1)}{8(d+2)}\frac{S_d}{(2\pi)^d}$$

Renormalization

Two poles should be eliminated → Two independent renormalization constants: $Z_{1,2} = 1 +$ poles in ε

➢ Renormalized parameters: $\nu_0 = \nu Z_\nu \quad g_0 = g M^{2\varepsilon} Z_g \quad \mu_0 = \mu Z_\mu$

$$Z_\nu = Z_1 \qquad Z_g = Z_1^{-3} \qquad Z_\mu = Z_2 Z_1^{-1}$$

Renormalized action:

$$S_R(\Phi) = \frac{1}{2} v' D_F v' + v'[-\partial_t v + \nu Z_1 \Delta v - (v \cdot \partial) v] + \varphi[-\partial_t \varphi + \mu \nu Z_2 \Delta \varphi - (v \cdot \partial) \varphi]$$

Analytic form for renormalization constants:

$$Z_1 = 1 - \frac{g}{\varepsilon} \frac{(d-1)}{8(d+2)} \frac{S_d}{(2\pi)^d} \qquad Z_2 = 1 - \frac{g}{\varepsilon} \frac{(d-1)}{4d(u+1)} \frac{S_d}{(2\pi)^d}$$

✓ Infrared fixed point (g_*, μ_*): $g_* \sim \varepsilon, \; g_* > 0, \quad \mu_* = const$

Conclusion

✓ Model with fully developed turbulence is studied by means of quantum field theory;

✓ The experimental Prandtl number is found to be in a range $< 0.7..0.9 >$. Further RG investigation of the model allows to estimate it's value, which is in good agreement with experiment [1].

[1] L. Ts. Adzhemyan, A. N. Vasil'ev, M. Gnatich, TMF, 58:1 (1984)