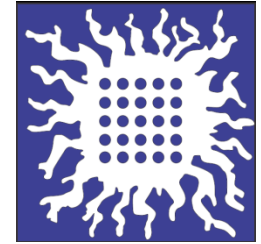




Cairo University



Studying of elastic Scattering in view of Optical Model

Milivoje Jojic¹, Magdy Ayad²

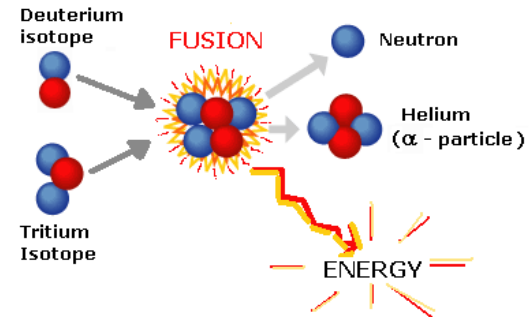
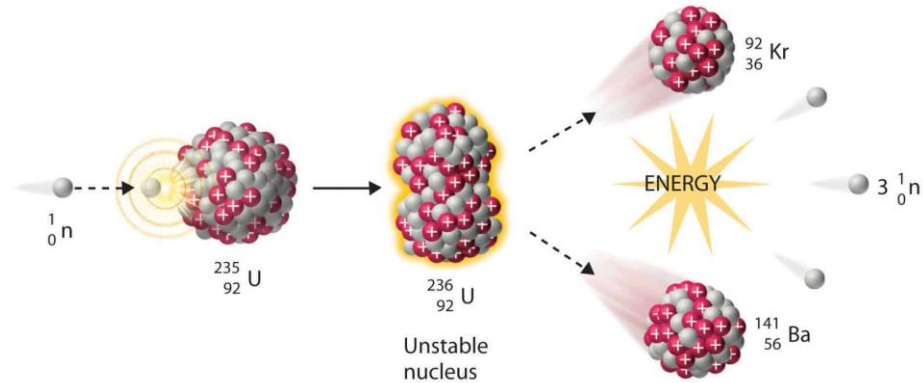
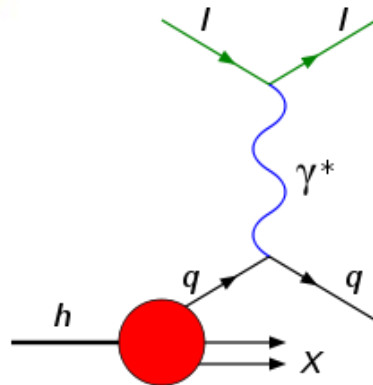
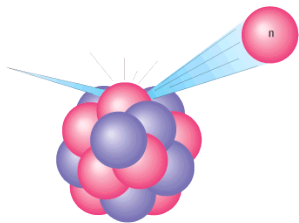
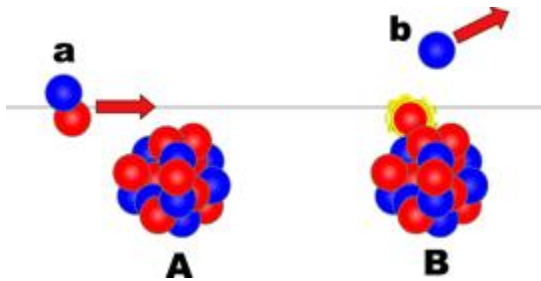
¹“Vinca” Institute for Nuclear Sciences, University of Belgrade, Serbia

² Cairo University

Supervisor : Dr. Vladimir Rachkov

---Types of nuclear interactions

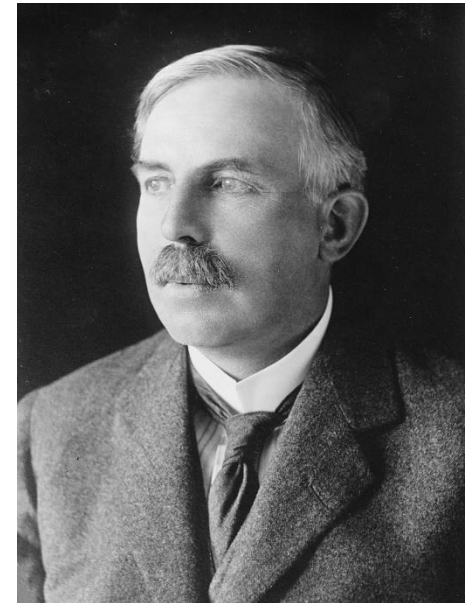
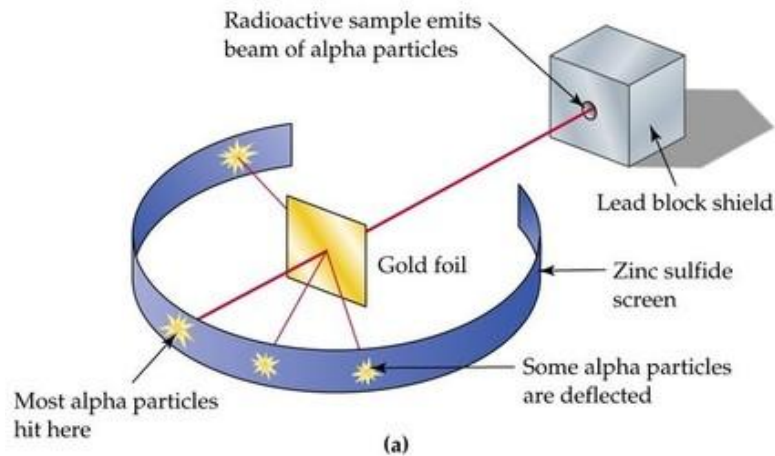
- **Elastic scattering** \longrightarrow Optical Model
- Inelastic scattering
- Transfer reaction
- Break-up reactions
- Fusion reaction
- Fission reaction ...



---a little bit of **History**

Rutherford scattering : Rutherford scattering is the elastic scattering of charged particles by the Coulomb interaction.

The classical Rutherford scattering process of alpha particles against gold foils which leads to discovery of atomic nucleus and a new model of the atom.



Ernest Rutherford

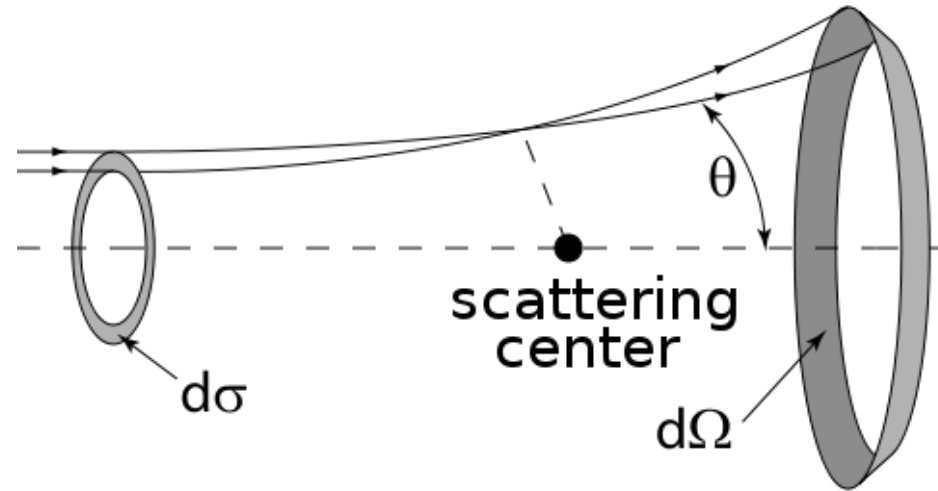
Inventor of the “art” of scattering

Cross Section

The number of particles entering a detector depends on:-

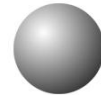
- flux of incident beam
- number of scattering centers in the target
- solid angular size of detector
- the cross section area for the reaction occur

$$\frac{\Delta N}{\Delta t} = j_i n \Delta \Omega \sigma \quad \frac{d\sigma}{d\Omega} = \frac{b}{\sin \theta} \left| \frac{db}{d\theta} \right|$$



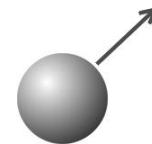
The total cross section of a hard sphere collision

$$\sigma_T = \pi r^2$$



The differential cross section of a hard sphere collision

$$\frac{d\sigma}{d\Omega} = \frac{R^2}{4}$$



Optical Model – main features

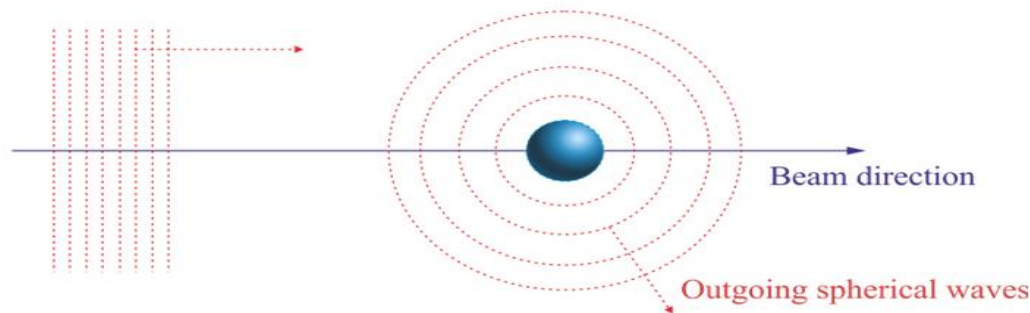
Schroedinger equation:

$$\left[-\frac{\hbar^2}{2\mu} \Delta + V(r) - E \right] \psi(r, \theta) = 0, \quad E > 0$$

Boundary conditions:

$$\psi(r = 0) = 0,$$

$$\psi(+\infty, \theta) = \exp(i \vec{k} \cdot \vec{r}) + f(\theta, \varphi) \frac{\exp(ikr)}{r}$$



Differential cross section:

$$\frac{d\sigma}{d\Omega} = |f(\theta, \varphi)|^2$$

Optical Model – main features

Identity:

$$\langle \vec{p}' | \hat{S} - 1 | \vec{p} \rangle = \frac{i}{2\pi m} \delta(E'_p - E_p) f(E_p, \theta, \varphi)$$

Let us calculate “S –1” matrix element:

$$\langle \vec{p}' | \hat{S} - 1 | \vec{p} \rangle = \frac{1}{\pi m} \delta(E'_p - E_p) \sum_{l=0}^{+\infty} \frac{2l+1}{4\pi} [s_l(E) - 1] P_l(\cos\theta)$$

Where $s_l(E)$ is defined as:

$$\langle E', l', m' | \hat{S} - 1 | E, l, m \rangle = \delta(E' - E) \delta_{l,l'} \delta_{m,m'} s_l(E)$$

Therefore holds:

$$f(E_p, \theta, \varphi) = \frac{1}{2ip} \sum_{l=0}^{+\infty} (2l+1) [s_l(E) - 1] P_l(\cos\theta)$$

Optical model potential

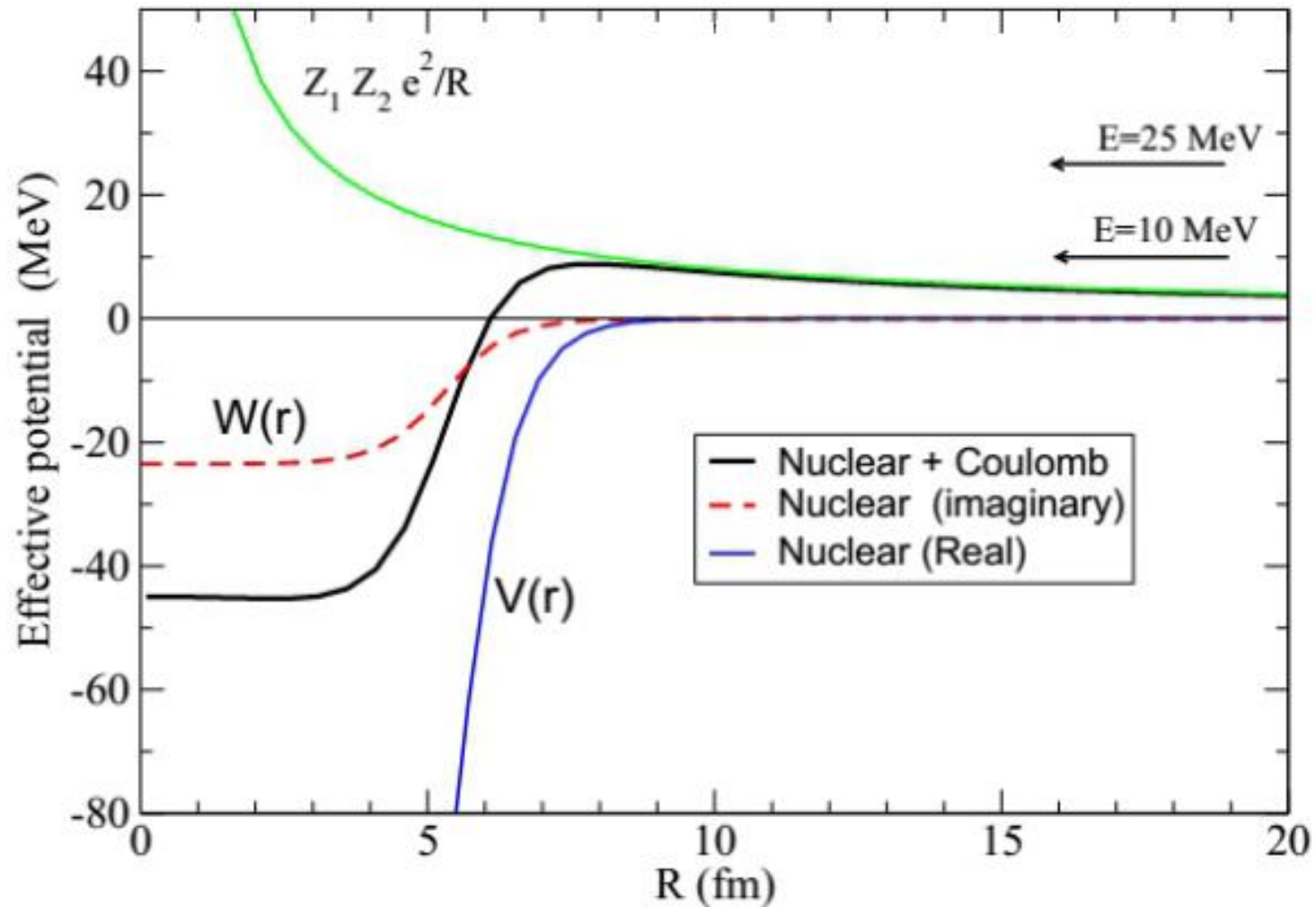
$$V_{OM} = V_c + V_{nucl} + V_{spin-orbit}$$

$$V_c = \begin{cases} \frac{Z_1 Z_2 e^2}{2R_c} \left(3 - \frac{r^2}{R_c^2} \right), & r < R_c; \\ \frac{Z_1 Z_2 e^2}{r}, & r > R_c; \end{cases}$$

$$V_{spin-orbit} = \vec{\sigma} \cdot \vec{L} \frac{V_{so}^0}{r} \frac{\partial f}{\partial r},$$
$$f = \left(1 + \frac{r - r_{so}}{a_{so}} \right)^{-1}$$

$$V_{nucl} = - \frac{V_0}{1 + \exp\left(\frac{r - R_v}{a_v}\right)} - i \frac{W_0}{1 + \exp\left(\frac{r - R_w}{a_w}\right)}$$

Effective interaction potential





NRV knowledge base

<http://nrv.jinr.ru>

The NRV web knowledge base is a unique interactive research system:

- Allows to run complicated computational codes
- Works in any internet browser
- Has graphical interface for preparation of input parameters and analysis of output results
- Combines computational codes with experimental databases on properties of nuclei and nuclear reactions
- Contains detailed description of models

The screenshot shows a web browser window displaying the NRV Nuclear Reactions Video Low Energy Nuclear Knowledge Base. The page features a blue header with the NRV logo and the text "Supported by Russian Foundation for Basic Research". Below the header is a grid of navigation links categorized into four main sections: Nuclear Properties, Nuclear Models, Nuclear Decays, and Nuclear Reactions. Each section contains sub-links to various models and codes. A "Getting Started" button with a green arrow is prominently displayed. A warning message about Java installation is also visible. The footer contains a disclaimer and a URL: nrv.jinr.ru/nrv/webnr/elastic_scattering/els1.htm.

Nuclear Properties	Nuclear Models	Nuclear Decays	Nuclear Reactions
Nuclear Map Systematics JS	Shell Model	Alpha - decay	Available beams Stable beams: EU Institutions, FLNR (Dubna) U400, FLNR (Dubna) U400M RIBs: GANIL, MSU
Getting Started	Liquid Drop Model	Beta - decay	Elastic scattering Classical Semiclassical Optical Model (Tutorial in Russian) Phase analysis
Warning: sections marked by this icon or without any icons require installation of Java and a Java-compatible browser Please, quote us as...	Two-Center Shell Model	Fission	Experimental Data Java JS $d\sigma/d\Omega$
		Decay of excited nuclei	Inelastic Scattering: DWBA model (DWUCK4 code) Adiabatic rotational model (FRESCO code) Coulomb excitation Direct process (DWBA) Channel coupling Deep inelastic collision
			Transfer reactions: Direct process (DWBA) JS JS Semiclassical approach (GRAZING code) 3-body classical model Two-nucleon transfer Massive transfer
			Fragmentation EPAX v.3 Break-up (DWBA) Semiclassical model
			Fusion Experimental

Measured cross section for a proton beam at different energies with a target of calcium (Ca 40)

Reference : J.F. Dicello Physical Review C , volume 4 Number 4 (1971) Page 1130

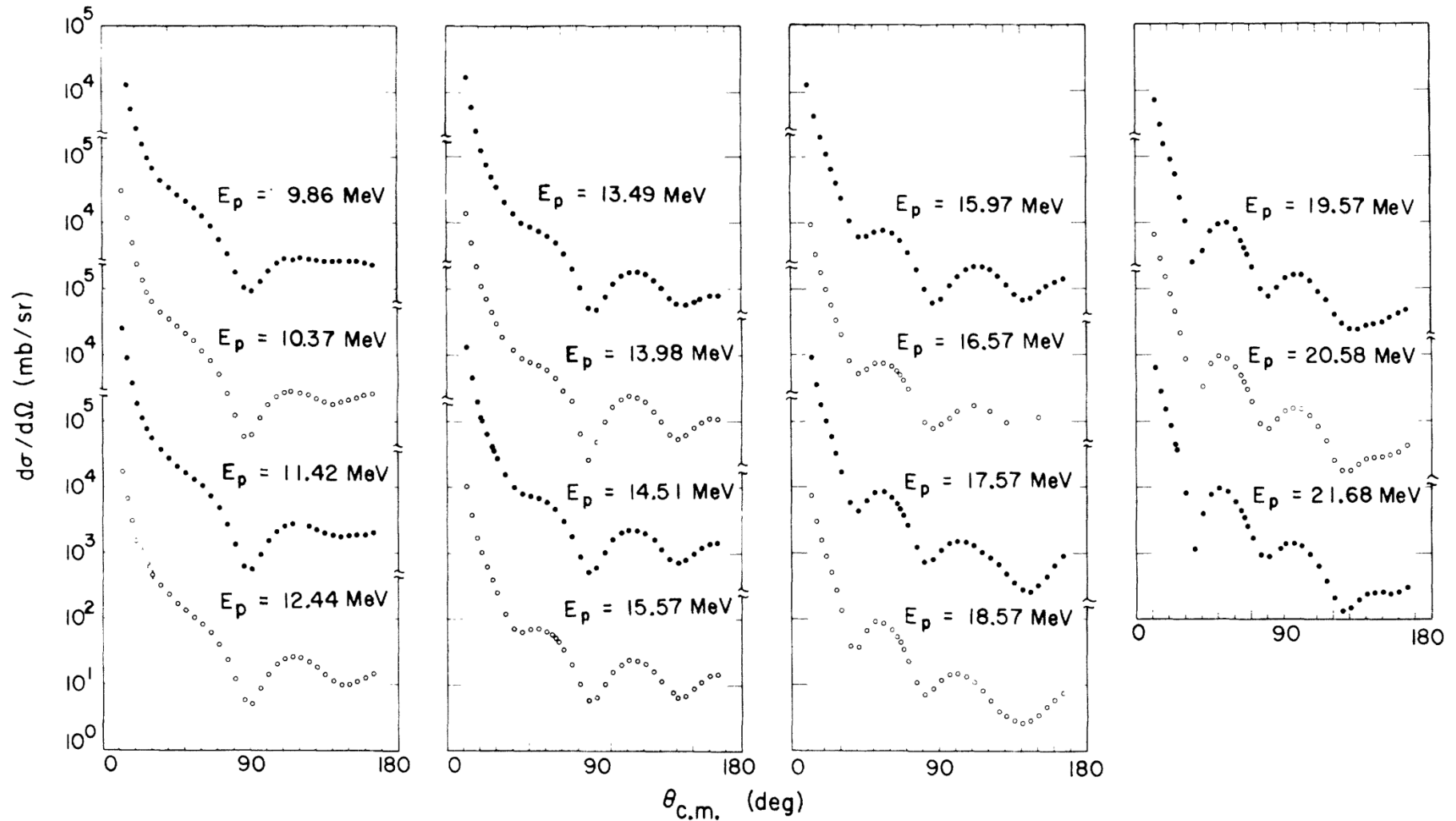


FIG. 4. Angular distributions. The differential elastic cross sections are shown as a function of angle in the center-of-mass system. The relative uncertainties are approximately the size of the symbols unless otherwise indicated.

NRV optical model

Quantum mechanical **Optical Model** code of the elastic scattering allows you to perform realistic calculations and to fit automatically all



Model Classical Optical NRV Description

Model Classical Optical NRV Description

Reaction

Projectile H r_0 fm R fm

Target Ca r_0 fm R fm

Energy MeV lab cm E/A

Experimental data

Potential forces

<input type="text" value="Superposition"/>	V_0^{vol}	<input type="text" value="-50.302"/>	MeV	r_0^{vol}	<input type="text" value="1.411"/>	fm	a^{vol}	<input type="text" value="0.187"/>	fm
	V_0^{sur}	<input type="text" value="-5.647"/>	MeV	r_0^{sur}	<input type="text" value="1.124"/>	fm	a^{sur}	<input type="text" value="0.158"/>	fm
Proximity b	r_0^{coul}	<input type="text" value="0.503"/>	fm						

N_{Re} N_{Im}

Absorptive pot.

<input type="text" value="Superposition"/>	W_0^{vol}	<input type="text" value="-0.355"/>	MeV	r_0^{vol}	<input type="text" value="0.267"/>	fm	a^{vol}	<input type="text" value="0.386"/>	fm
	W_0^{sur}	<input type="text" value="-6.427"/>	MeV	r_0^{sur}	<input type="text" value="1.391"/>	fm	a^{sur}	<input type="text" value="0.482"/>	fm

Spin-orbit interaction

Spin 0 1/2 V_0 MeV W_0 MeV r_0 fm a fm

Java blocked?

Real part Radius Radius of the Coulomb Potential
 Imaginary part Diffuseness Folding potential
 N_{Re} N_{Im}
 Maximal number of fit steps Stop, when change is less than %

NRV optical model

NRV: Optical Model of Ela... X +

nrvjnr.ru/nrv/webnrv/elastic_scattering/dialog.php?COMMAND=CALCULATE&Z_PROJ=1&A_PROJ=1&R_PROJ=1.2&DEF_PRO

NRV: Optical Model
Elastic scattering of $^1\text{H} + ^{40}\text{Ca}$ at $E/A = 9.86$ MeV

Optical Model parameters

Coulomb $r_0(R)$, fm 0.503 (1.72)	Real part			Imaginary part		
	V_0 , MeV	$r_0(R)$, fm	a , fm	W_0 , MeV	$r_0(R)$, fm	a , fm
Volume	-50.302	1.411 (4.826)	0.187	-0.355	0.287 (0.913)	0.388
Surface	-5.847	1.124 (3.844)	0.158	-8.427	1.372 (4.692)	0.497
Spin-Orbit	0.328	0.112 (0.383)	0.805	-19.387	0.112 (0.383)	0.805
Proximity						
Folding						

	Before fitting	After fitting
σ_R , mb	520.01	-
σ_{tot} , mb	1310.23	-
χ^2 / N points		-

Other quantities

$E_{lab} = 9.86$ MeV
 $E_{cm} = 9.62$ MeV
 $k = 0.67$ fm $^{-1}$
 $\eta = 1.003$
 $R_{max} = 17.08$ fm
 $dr = 0.2$ fm

Fitting process

$N_{steps} = \text{no fit}$
 $\Delta\chi^2/\chi^2 = \text{no fit}$
 Use obtained OMP in
 DWBA: Inelastic scattering (exit channel)

Submit

OM Interaction

S-matrix

Diff. Cross Section

Elastic Scattering: p+40Ca, Ecm = 9.6 MeV

File View Other plots Dependence on ...

Click on a plot to process it

NRV optical model

NRV NRV: Optical Model of Ela... X +

nrvjin.ru/nrv/webnrv/elastic_scattering/dialog.php?COMMAND=CALCULATE&Z_PROJ=1&A_PROJ=1&R_PROJ=1.2&DEF_PRO

NRV: Optical Model
Elastic scattering of $^1\text{H} + ^{40}\text{Ca}$ at $E/A = 9.86$ MeV

Optical Model parameters

	Real part			Imaginary part		
	V_0 , MeV	$r_0(R)$, fm	a , fm	W_0 , MeV	$r_0(R)$, fm	a , fm
Coulomb $r_0(R)$, fm 0.503 (1.72)						
Volume	-50.302	1.411 (4.826)	0.187	-0.355	0.287 (0.913)	0.388
Surface	-5.647	1.124 (3.844)	0.158	-8.427	1.372 (4.692)	0.497
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$E_{lab} = 9.86$ MeV
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 $k = 0.67$ fm $^{-1}$
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 $R_{max} = 17.08$ fm
 $dr = 0.2$ fm

	Before fitting	After fitting
σ_R , mb	520.01	-
σ_{tot} , mb	1310.23	-
χ^2 / N points		-

Fitting process

$N_{steps} = \text{no fit}$
 $\Delta\chi^2/\chi^2 = \text{no fit}$
 Use obtained OMP in
 DWBA: Inelastic scattering (exit channel)

Submit

OM Interaction

S-matrix

Diff. Cross Section

OM interaction

File Parameters

V(r): Volume + Surface
 $V_0 = -50.30$ MeV $V_0 = -5.65$ MeV
 $r_0 = 1.41$ (R=4.83) fm $r_0 = 1.12$ (R=3.84) fm
 $a_V = 0.19$ fm $a_V = 0.16$ fm

W(r): Volume + Surface
 $W_0 = -0.35$ MeV $W_0 = -6.43$ MeV
 $r_0 = 0.27$ (R=0.91) fm $r_0 = 1.37$ (R=4.69) fm
 $a_W = 0.39$ fm $a_W = 0.50$ fm

Click on a plot to process it

NRV optical model

NRV: Optical Model of Ela... X +

nrvjinn.ru/nrv/webnrv/elastic_scattering/dialog.php?COMMAND=CALCULATE&Z_PROJ=1&A_PROJ=1&R_PROJ=1.2&DEF_PRO

Search

NRV: Optical Model
Elastic scattering of $^1\text{H} + ^{40}\text{Ca}$ at $E/A = 9.86$ MeV

Optical Model parameters

Coulomb $r_0(R)$, fm 0.503 (1.72)	Real part			Imaginary part		
	V_0 , MeV	$r_0(R)$, fm	a , fm	W_0 , MeV	$r_0(R)$, fm	a , fm
Volume	-50.302	1.411 (4.826)	0.187	-0.355	0.267 (0.913)	0.388
Surface	-5.647	1.124 (3.844)	0.158	-6.427	1.372 (4.692)	0.497
Spin-Orbit	0.326	0.112 (0.383)	0.805	-19.387	0.112 (0.383)	0.805
Proximity						
Folding						

Other quantities

$E_{lab} = 9.86$ MeV
 $E_{cm} = 9.82$ MeV
 $k = 0.67$ fm $^{-1}$
 $\eta = 1.003$
 $R_{max} = 17.08$ fm
 $dr = 0.2$ fm

Fitting process

Nsteps = no fit
 $\Delta\chi^2/\chi^2 = \text{no fit}$
 Use obtained OMP in
 DWBA: Inelastic scattering (exit channel)

	Before fitting	After fitting
σ_R , mb	520.01	-
σ_{tot} , mb	1310.23	-
χ^2 / N points		-

OM Interaction

S-matrix

Diff. Cross Section

Click on a plot to process it

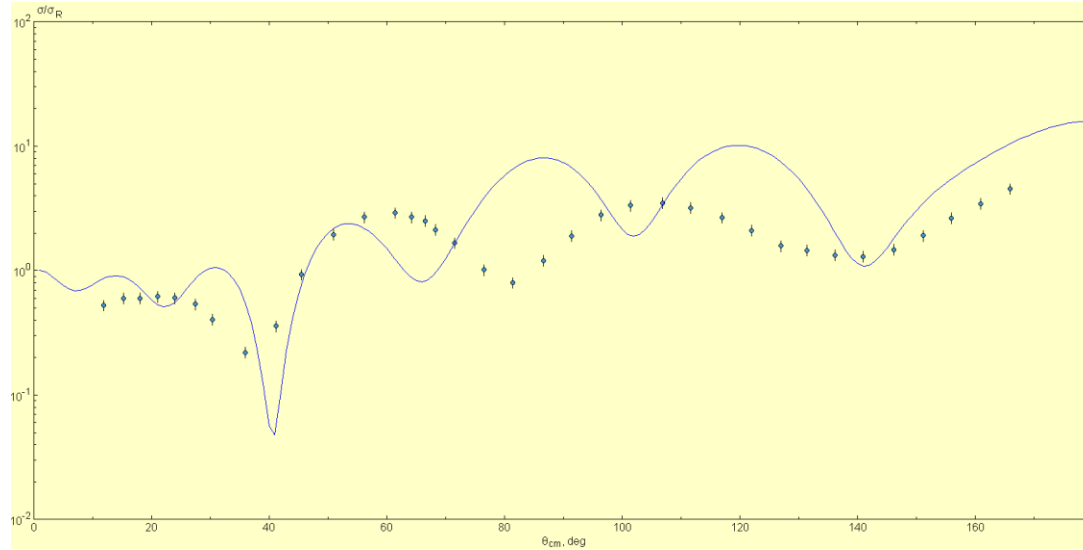
Partial S-matrix

File View

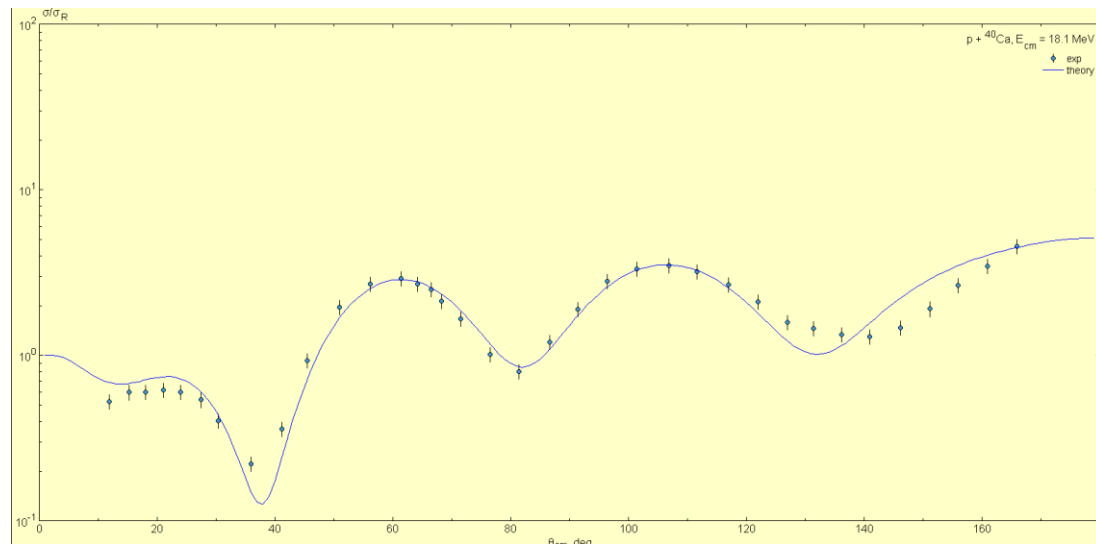
Legend: $|S_L^{(+)}|$ (black squares), $|S_L^{(-)}|$ (blue circles)

Elastic scattering of $p+^{40}\text{Ca}$

Before fitting

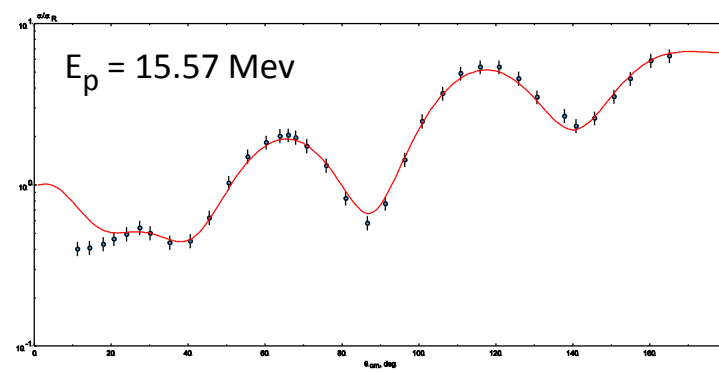
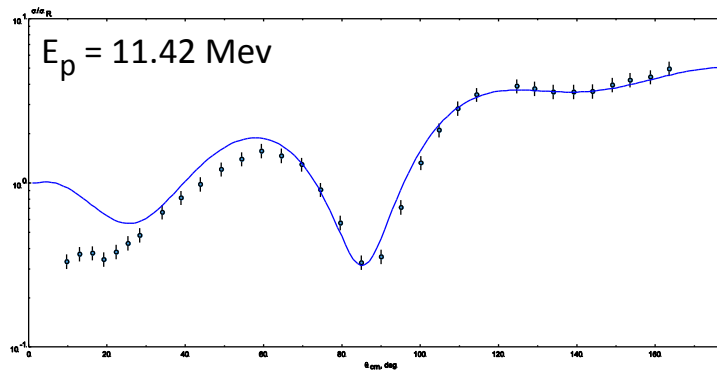
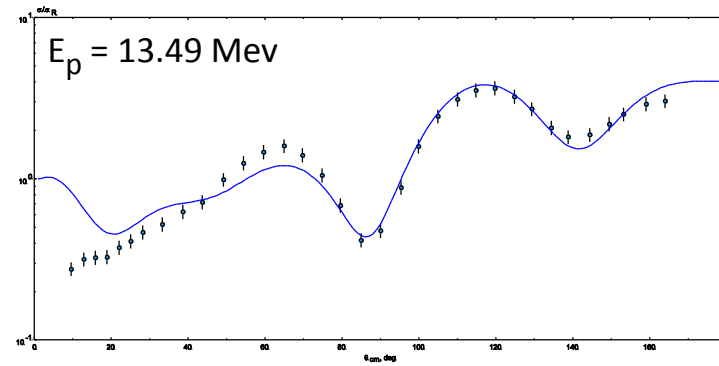
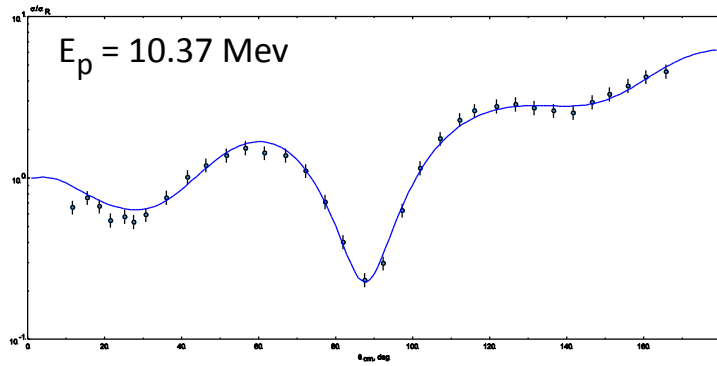
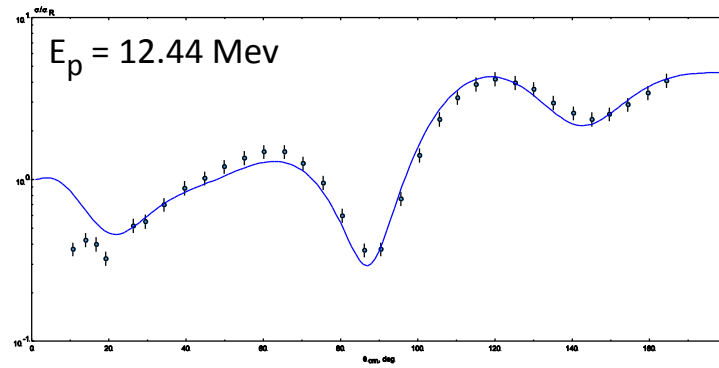
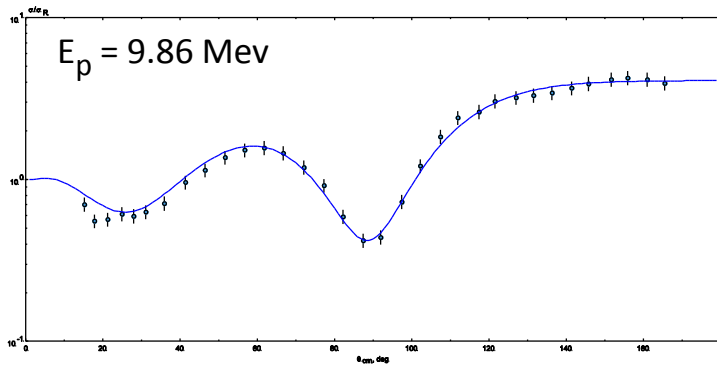


After fitting



Results

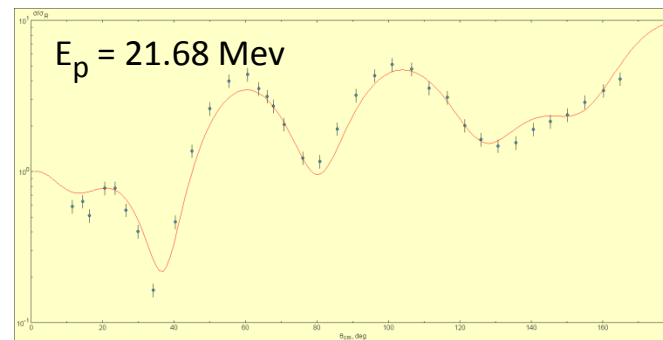
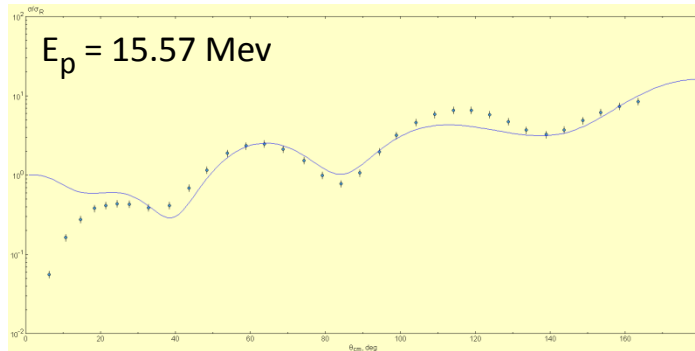
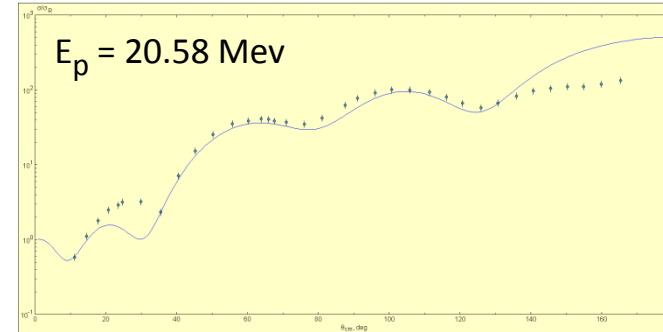
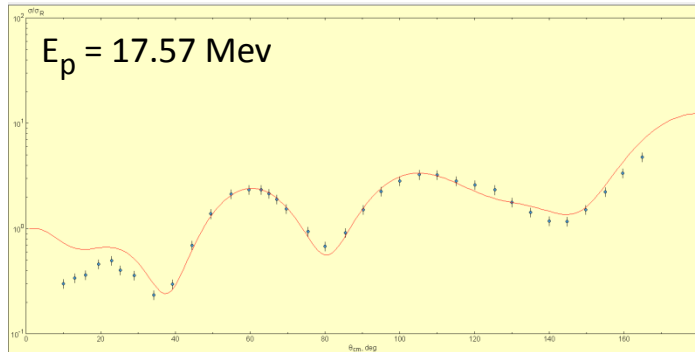
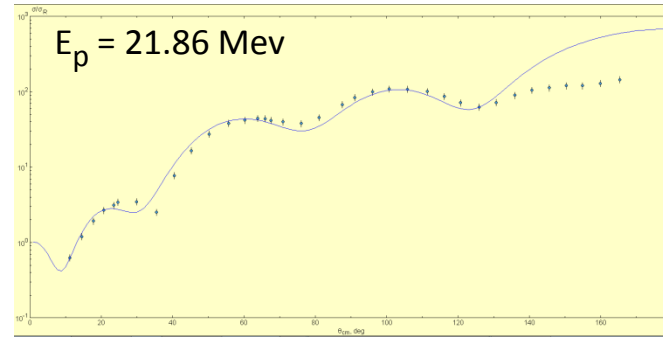
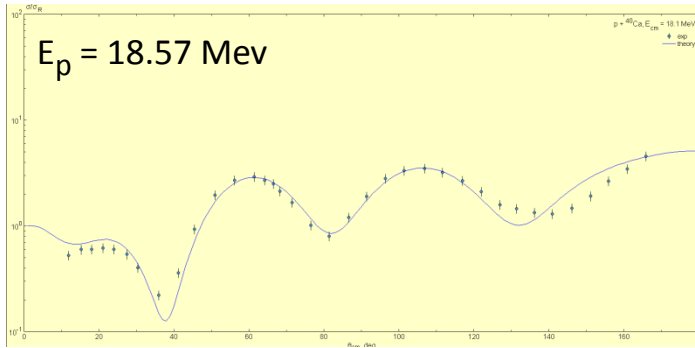
σ/σ_R



θ deg

Results

σ/σ_R



θ deg

Summary

- We studied basic aspects of the elastic scattering theory and the Optical Model.
- By choosing appropriate values of the parameters of the potential, we were able to construct a successful description of the cross section.
- Knowledge obtained in OM study is applicable for studying other non-elastic channels

Thanks for your attention.

Questions ?