Studying of elastic Scattering in view of Optical Model

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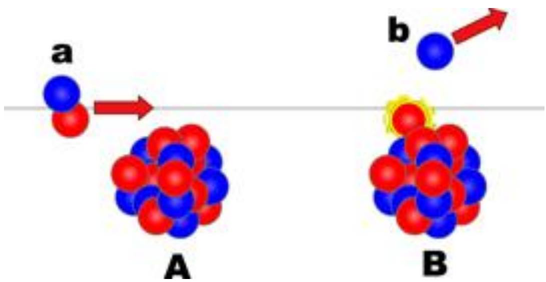
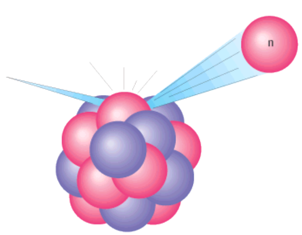
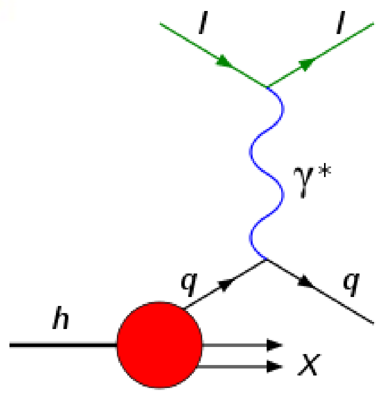
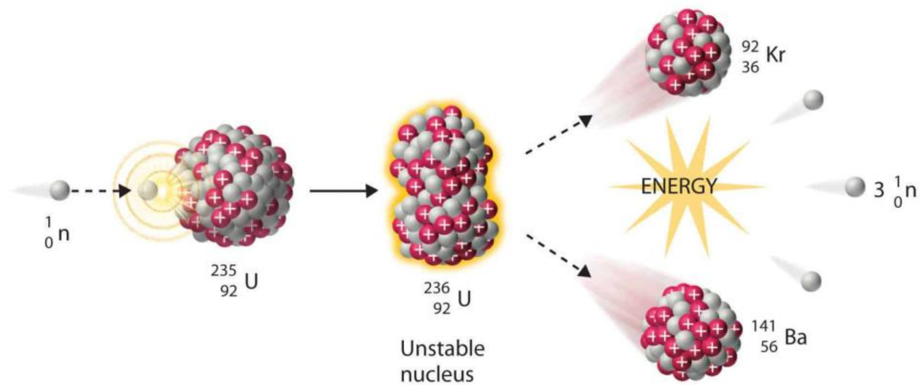
---Types of nuclear interactions

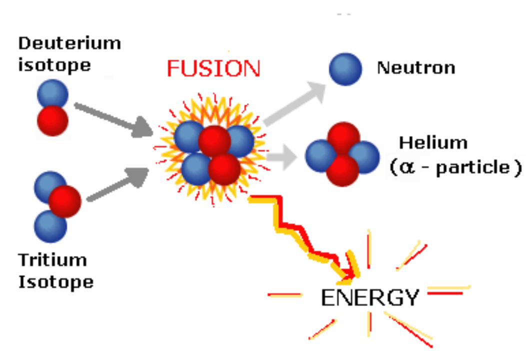
- **Elastic scattering** → **Optical Model**
- **Inelastic scattering**
- **Transfer reaction**
- **Break-up reactions**
- **Fusion reaction**
- **Fission reaction …**





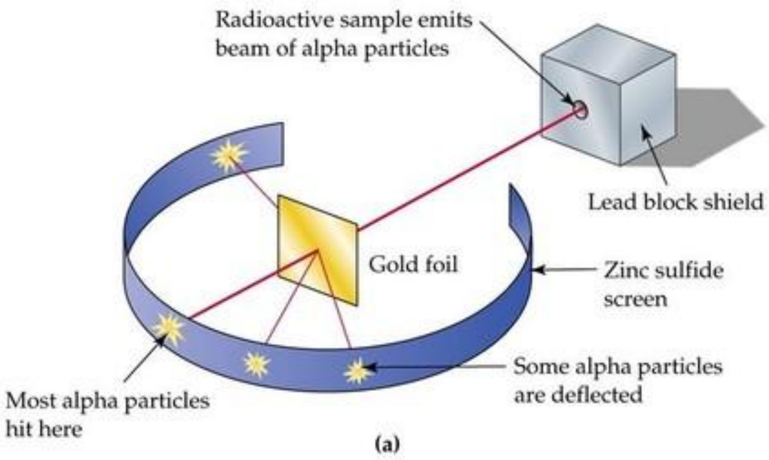


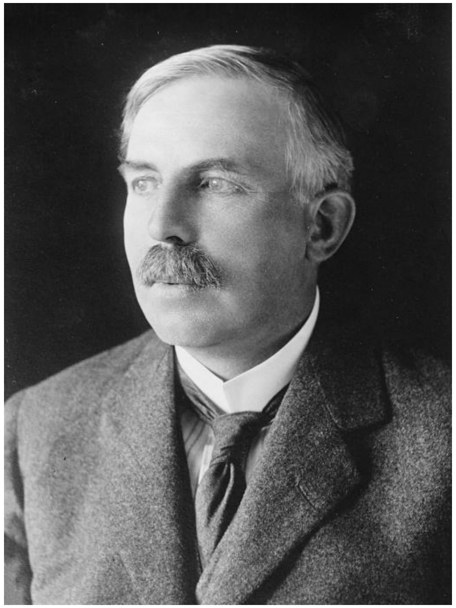




---a little bit of History

Rutherford scattering : Rutherford scattering is the elastic scattering of charged particles by the Coulomb interaction.
The classical Rutherford scattering process of alpha particles against gold foils which leads to discovery of atomic nucleus and a new model of the atom.



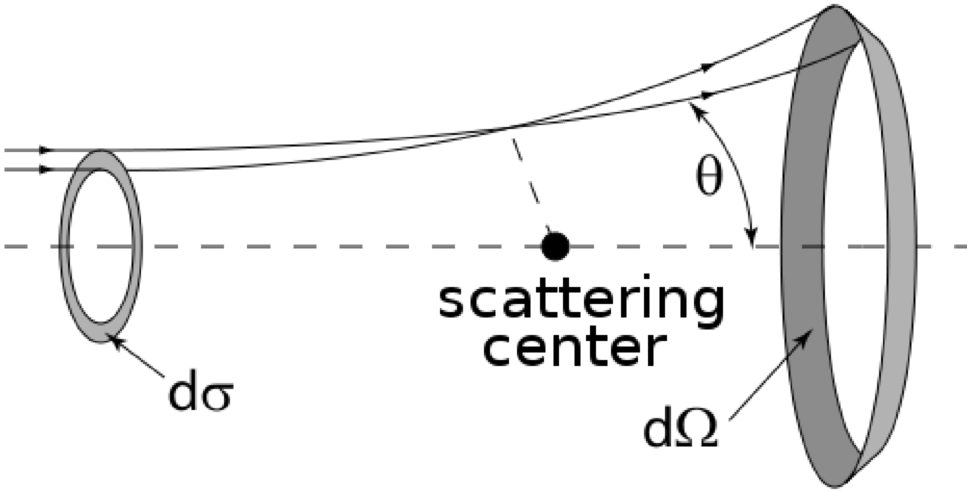


Ernest Rutherford
Inventor of the "art" of scattering

Cross Section

The number of particles entering a detector depends on:-

- flux of incident beam
- number of scattering centers in the target
- solid angular size of detector
- the cross section area for the reaction occur

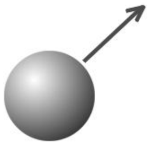


$$\frac{\Delta N}{\Delta t} = j_i \, n \, \Delta\Omega \, \sigma \qquad \frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$

The total cross section of a hard sphere collision

$$\sigma_T = \pi r^2$$

The differential cross section of a hard sphere collision



$$\frac{d\sigma}{d\Omega} = \frac{R^2}{4}$$

Optical Model – main features

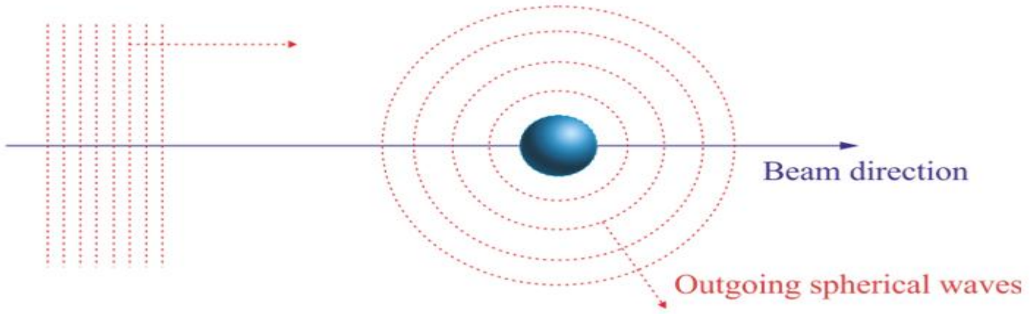
Schroedinger equation:

$$\left[-\frac{\hbar^2}{2\mu}\Delta + V(r) \;- E\right]\psi(r,\theta) = 0\,,\; E > 0$$

Boundary conditions:

$$\psi(r = 0) = 0,$$

$$\psi(+\infty,\theta) = \exp\left(i\,\vec{k}\cdot\vec{r}\right) + f(\theta,\varphi)\frac{\exp(ikr)}{r}$$



Differential cross section: $$\frac{d\sigma}{d\Omega} = \;|f(\theta,\varphi)|^2$$

Optical Model – main features

Identity:

$$\langle \vec{p}' | \hat{S} - 1 | \vec{p} \rangle = \frac{i}{2\pi m} \delta\left(E'_p - E_p\right) f(E_p, \theta, \varphi)$$

Let us calculate "S –1" matrix element:

$$\langle \vec{p}' | \hat{S} - 1 | \vec{p} \rangle = \frac{1}{\pi m} \delta\left(E'_p - E_p\right) \sum_{l=0}^{+\infty} \frac{2l+1}{4\pi} [s_l(E) - 1] P_l(cos\theta)$$

Where $s_l(E)$ is defined as:

$$\langle E', l', m' | \hat{S} - 1 | E, l, m \rangle = \delta(E' - E) \delta_{l,l'} \delta_{m,,m'} s_l(E)$$

Therefore holds:

$$f(E_p, \theta, \varphi) = \frac{1}{2ip} \sum_{l=0}^{+\infty} (2l+1)[s_l(E) - 1] P_l(cos\theta)$$

Optical model potential

$$V_{OM} = V_c + V_{nucl} + V_{spin-orbit}$$

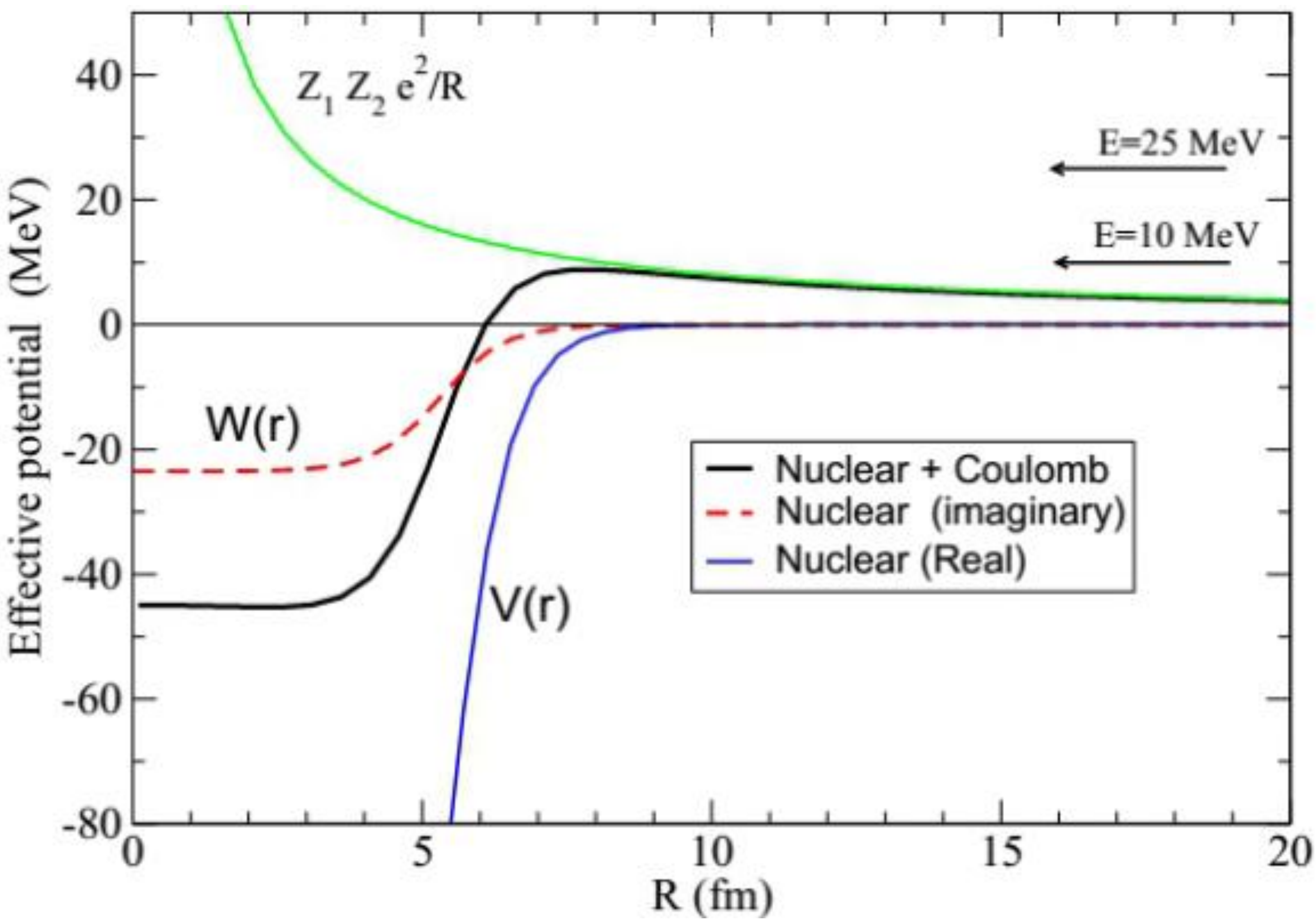
$$V_c = \begin{cases} \frac{Z_1 Z_2 e^2}{2R_C}\left(3 - \frac{r^2}{{R_C}^2}\right), \ r < R_C ; \\ \frac{Z_1 Z_2 e^2}{r}, \ r > R_C ; \end{cases}$$

$$V_{spin-orbit} = \vec{\sigma} \cdot \vec{L} \frac{{V_{so}}^0}{r} \frac{\partial f}{\partial r},$$

$$f = \left(1 + \frac{r - r_{so}}{a_{so}}\right)^{-1}$$

$$V_{nucl} = -\frac{V_0}{1+exp\left(\frac{r-R_v}{a_v}\right)} - i \ \frac{W_0}{1+\exp\left(\frac{r-R_W}{a_W}\right)}$$

Effective interaction potential

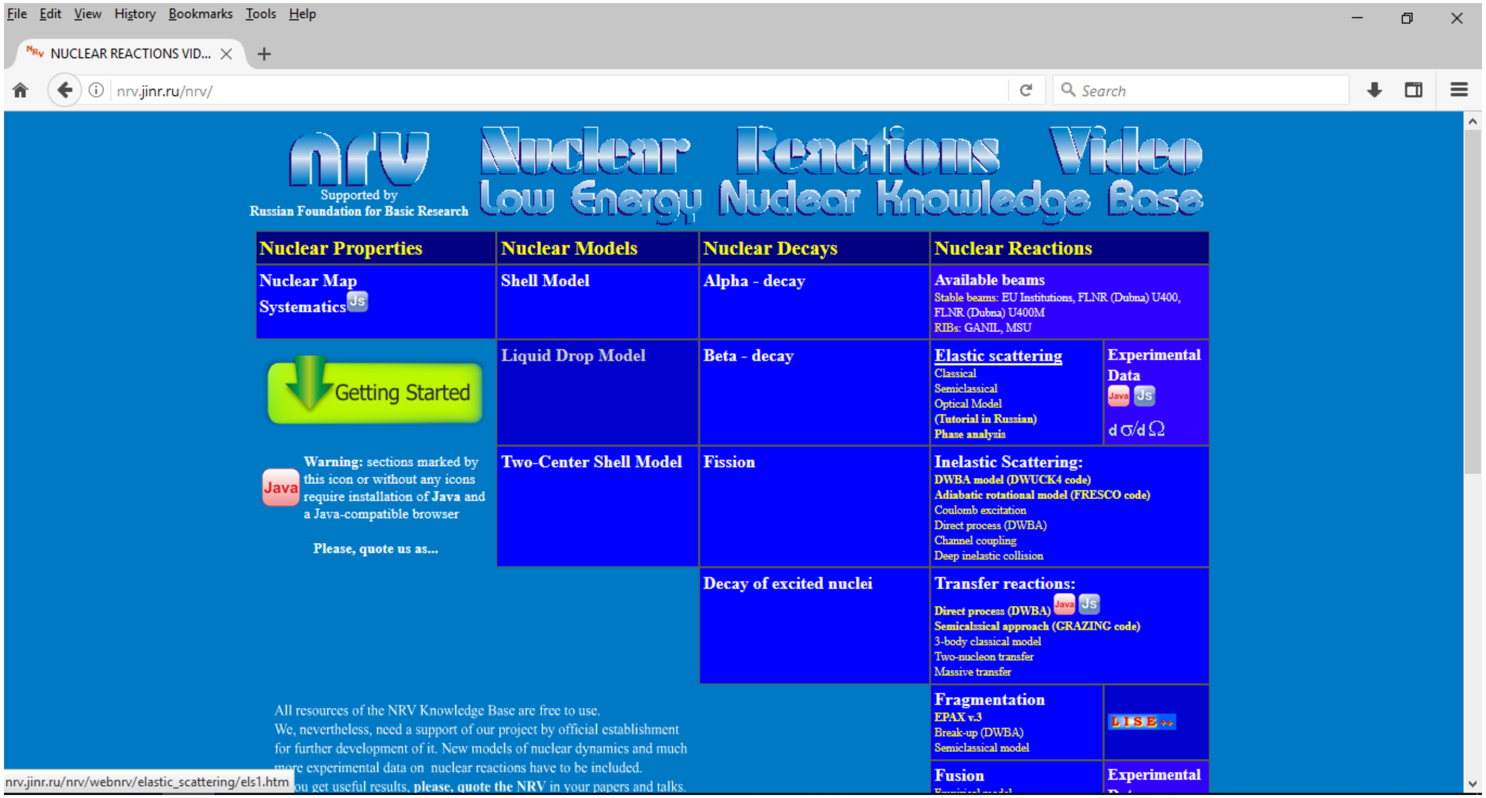


NRV knowledge base

http://nrv.jinr.ru

The NRV web knowledge base is a unique interactive research system:

- Allows to run complicated computational codes
- Works in any internet browser
- Has graphical interface for preparation of input parameters and analysis of output results
- Combines computational codes with experimental databases on properties of nuclei and nuclear reactions
- Contains detailed description of models



Measured cross section for a proton beam at different energies with a target of calcium (Ca 40)

Reference : J.F. Dicello Physical Review C , volume 4 Number 4 (1971) Page 1130

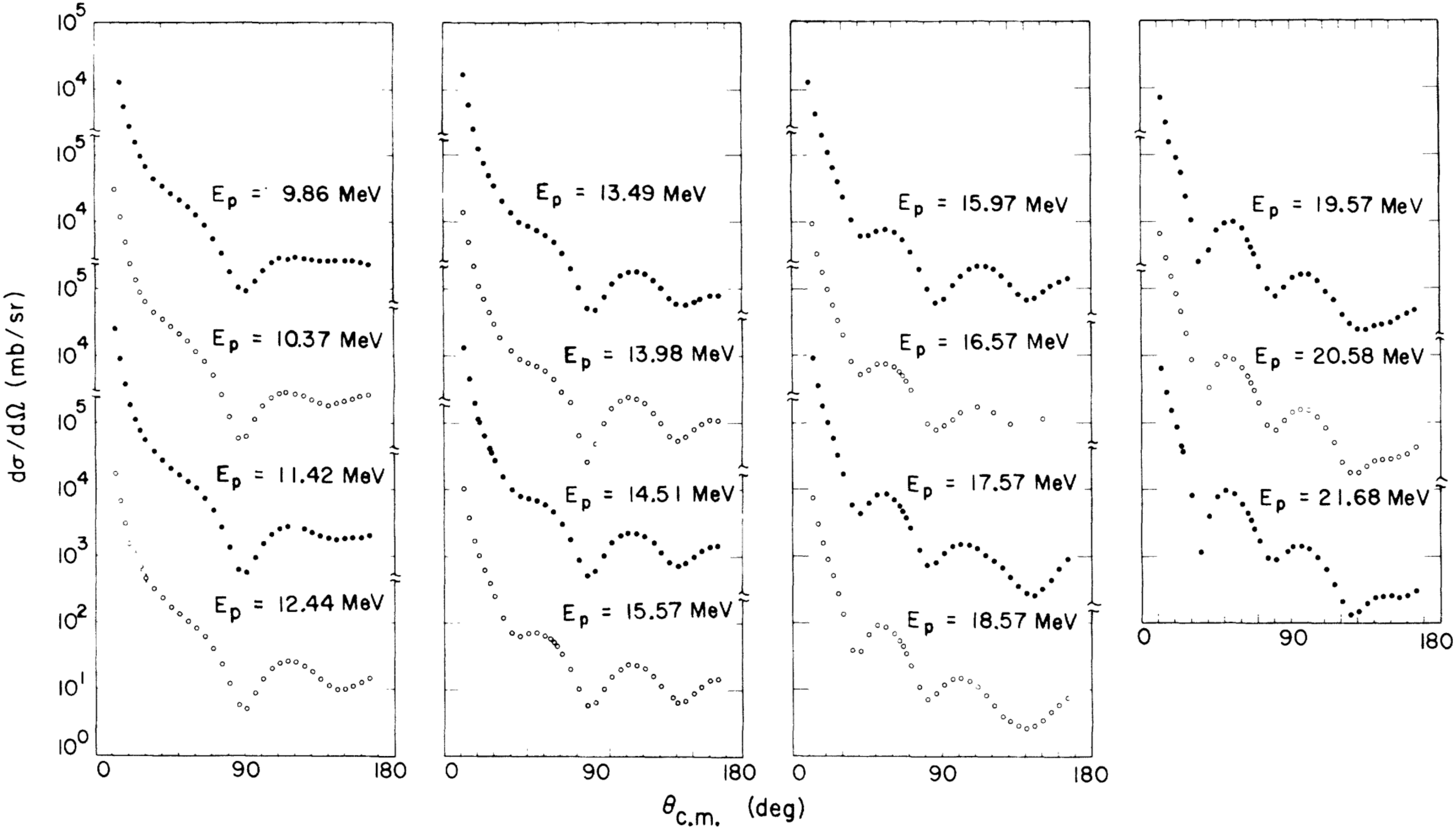
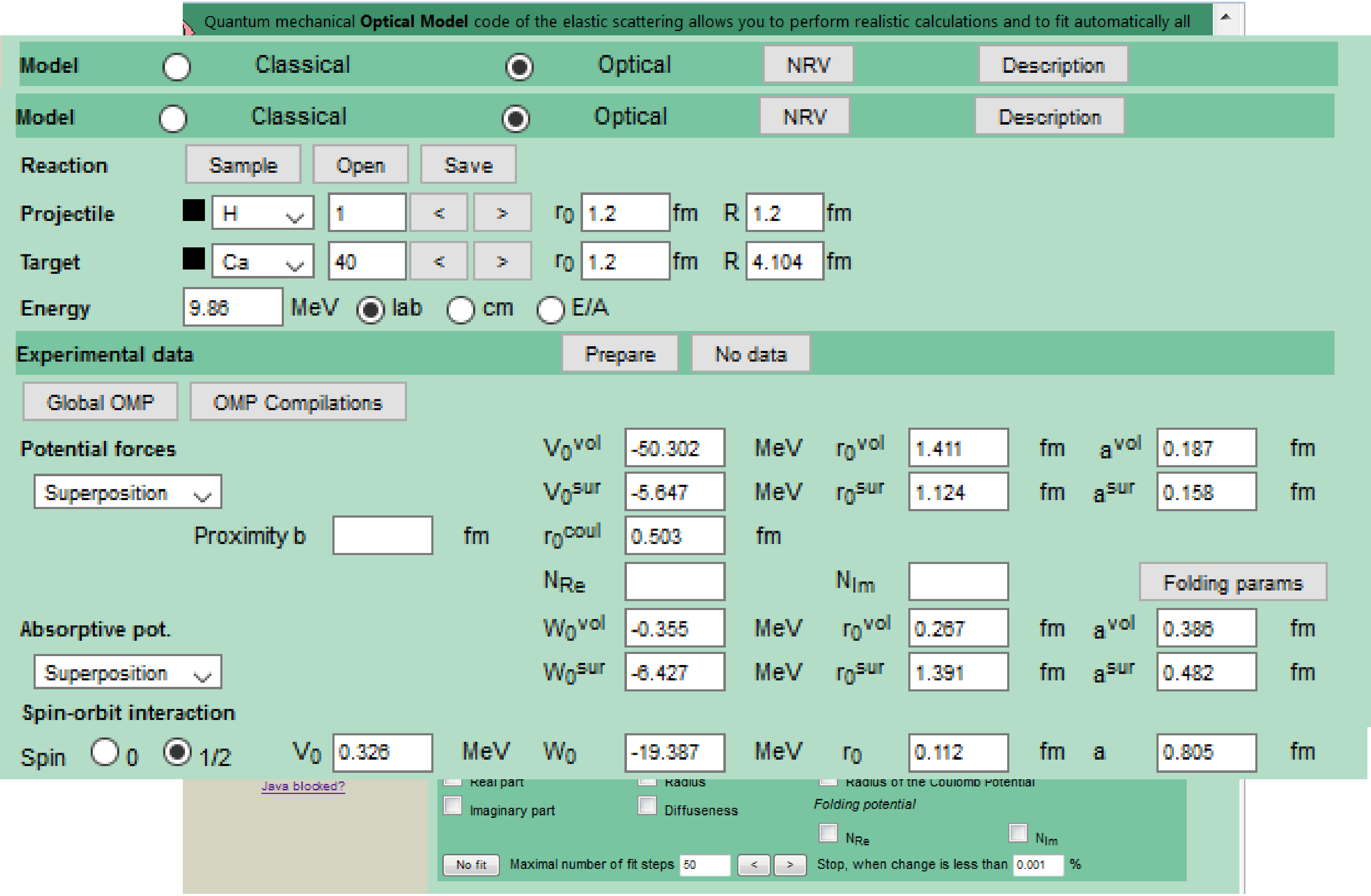
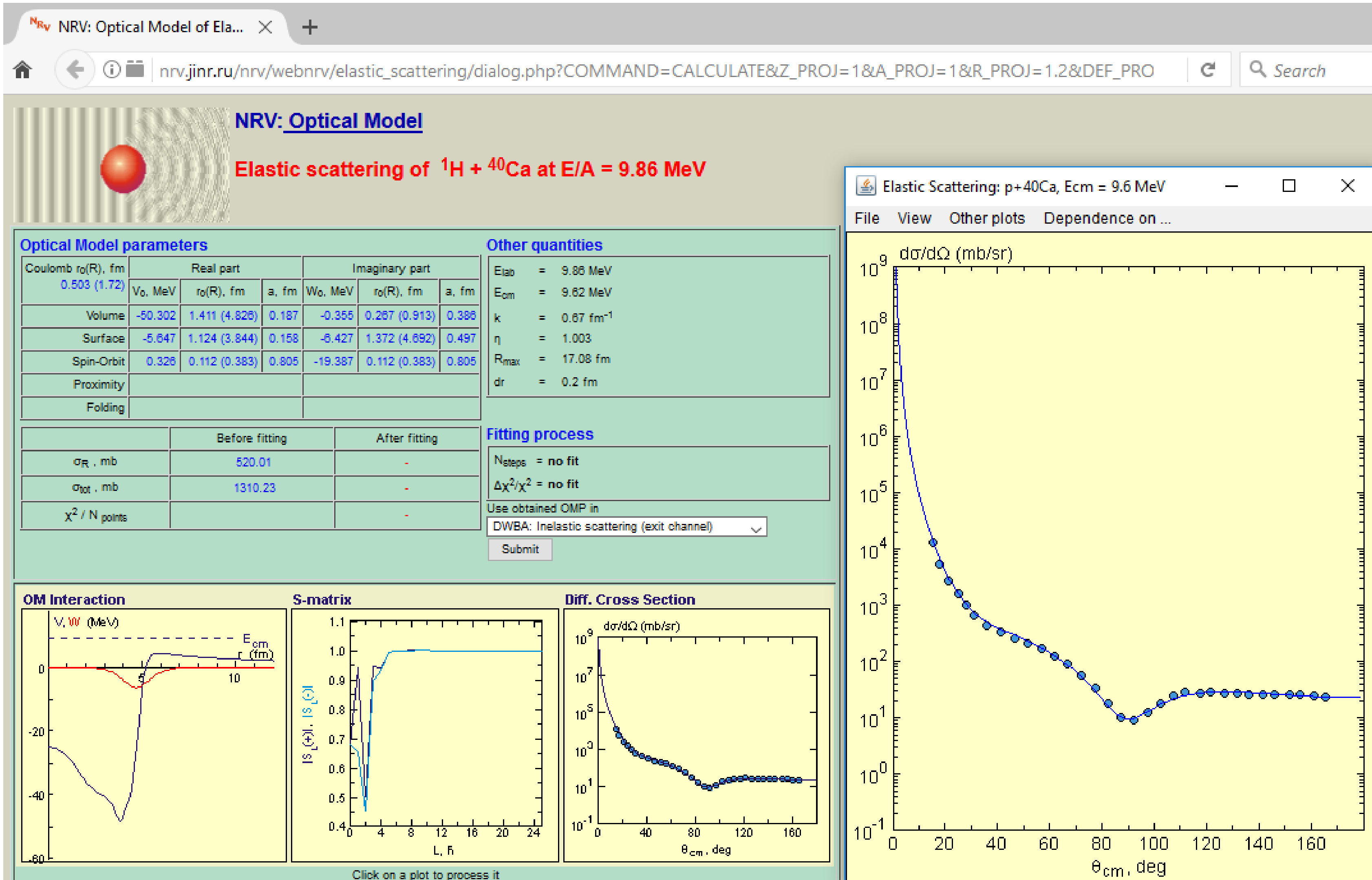


FIG. 4. Angular distributions. The differential elastic cross sections are shown as a function of angle in the center-of-mass system. The relative uncertainties are approximately the size of the symbols unless otherwise indicated.

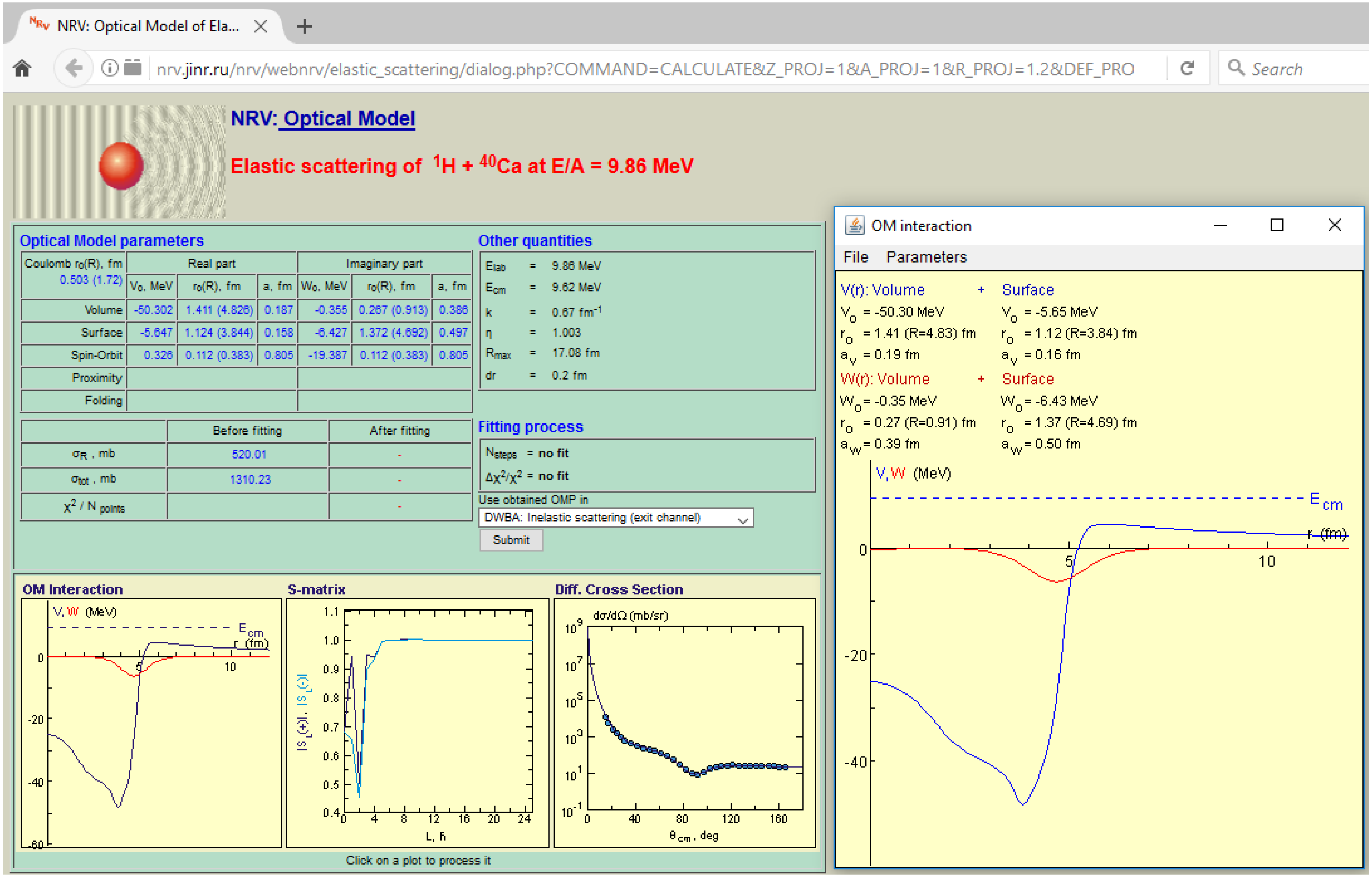
NRV optical model



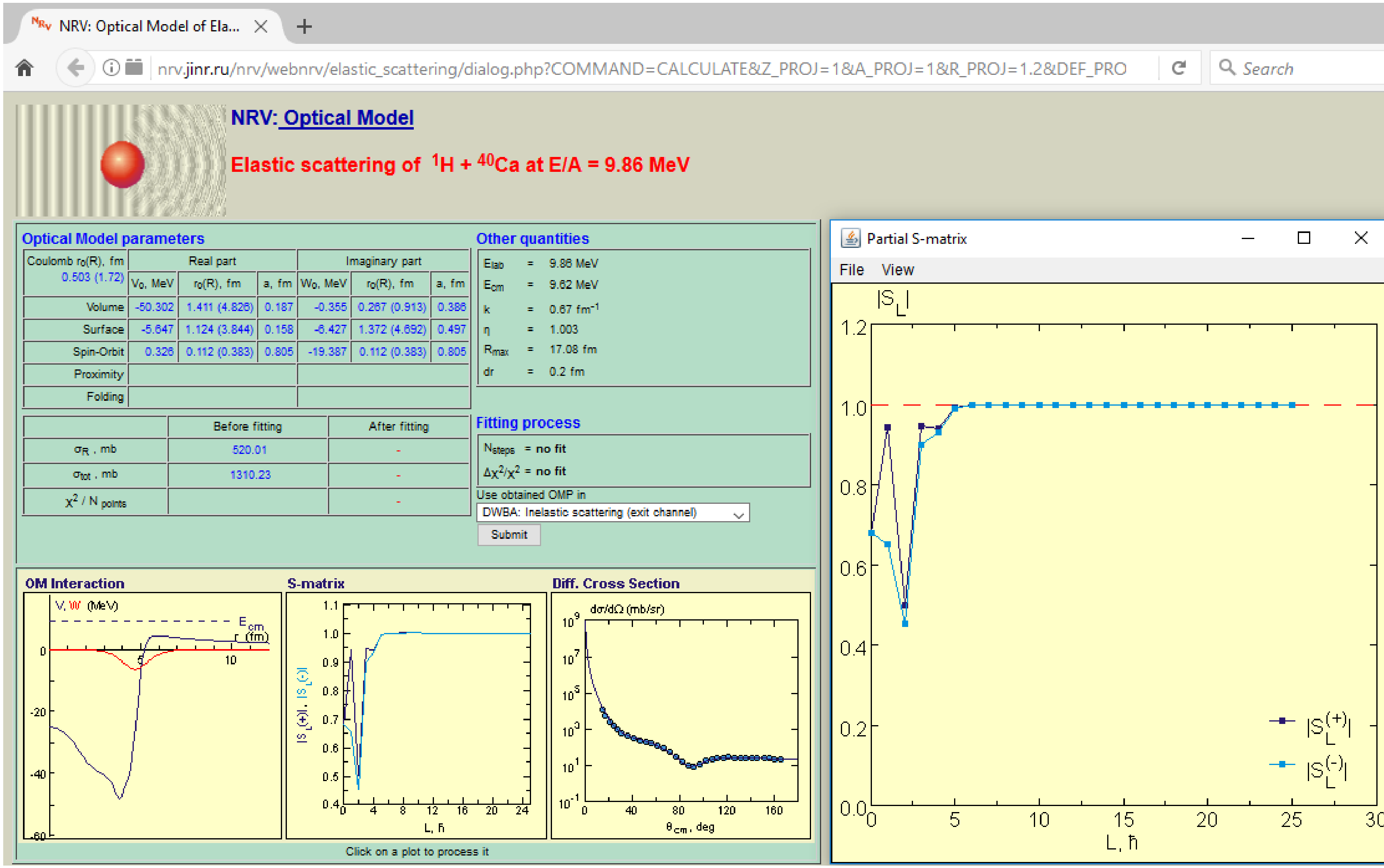
NRV optical model



NRV optical model

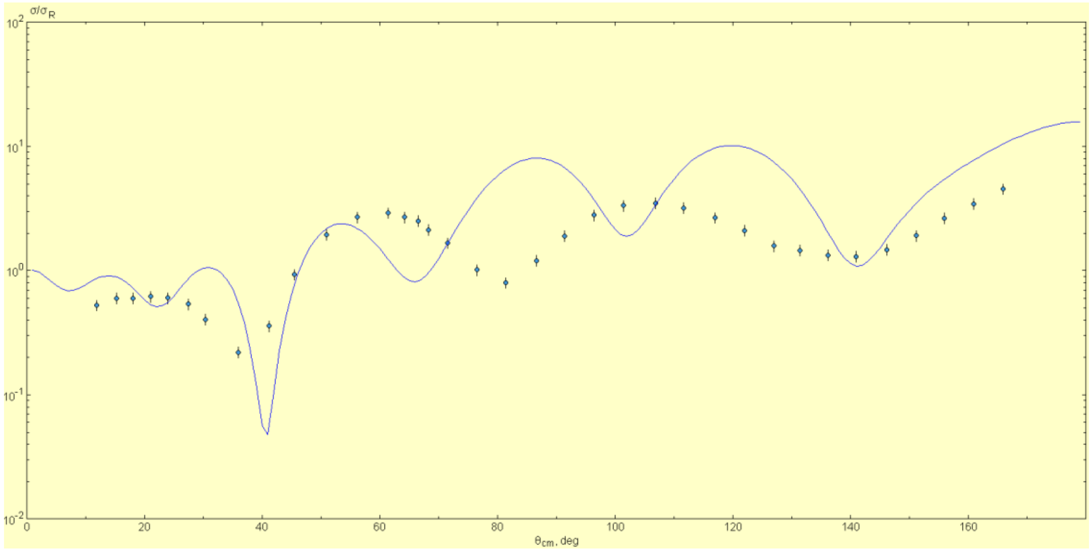


NRV optical model

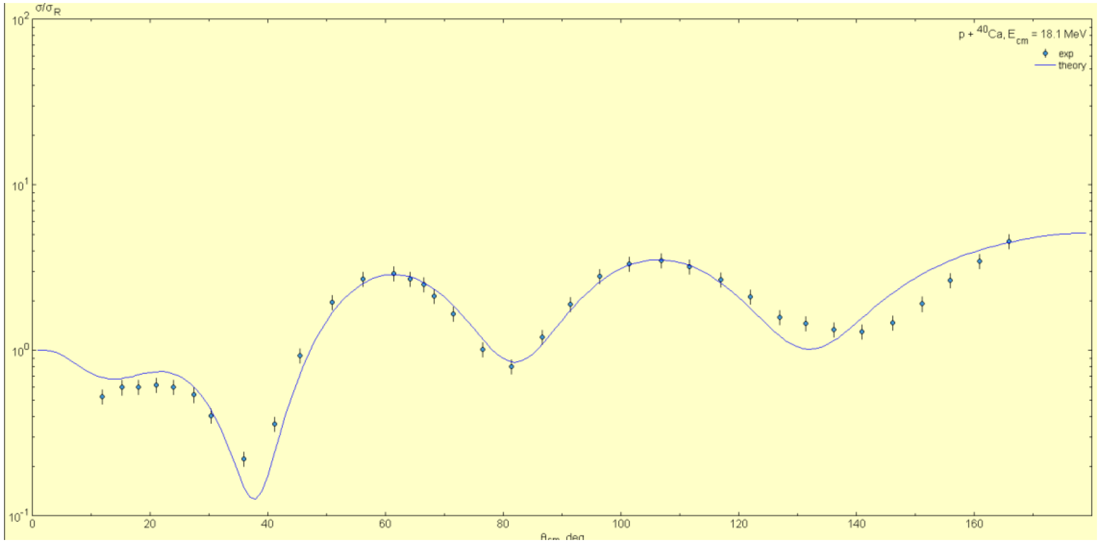


Elastic scattering of p+^{40}Ca

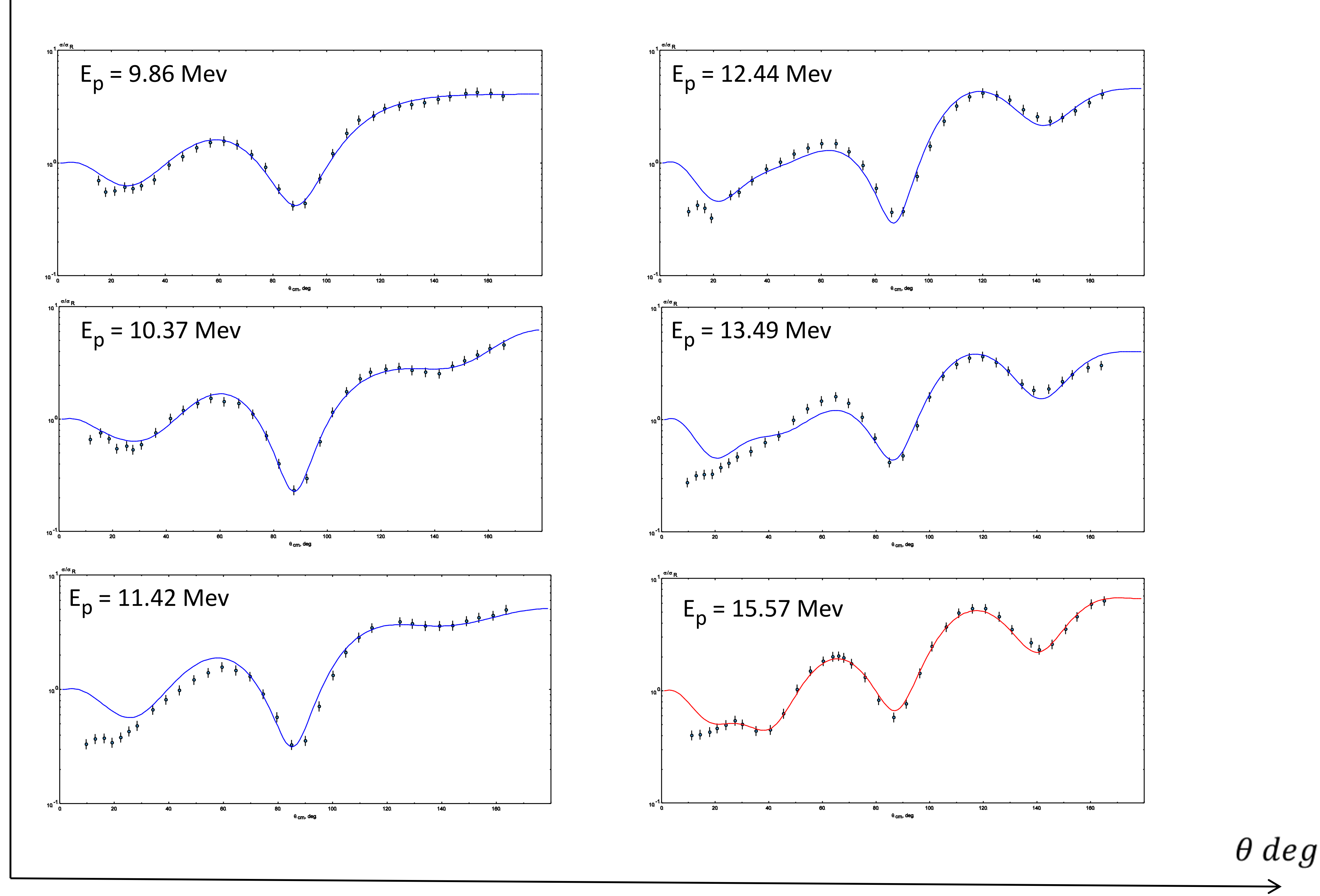
Before fitting



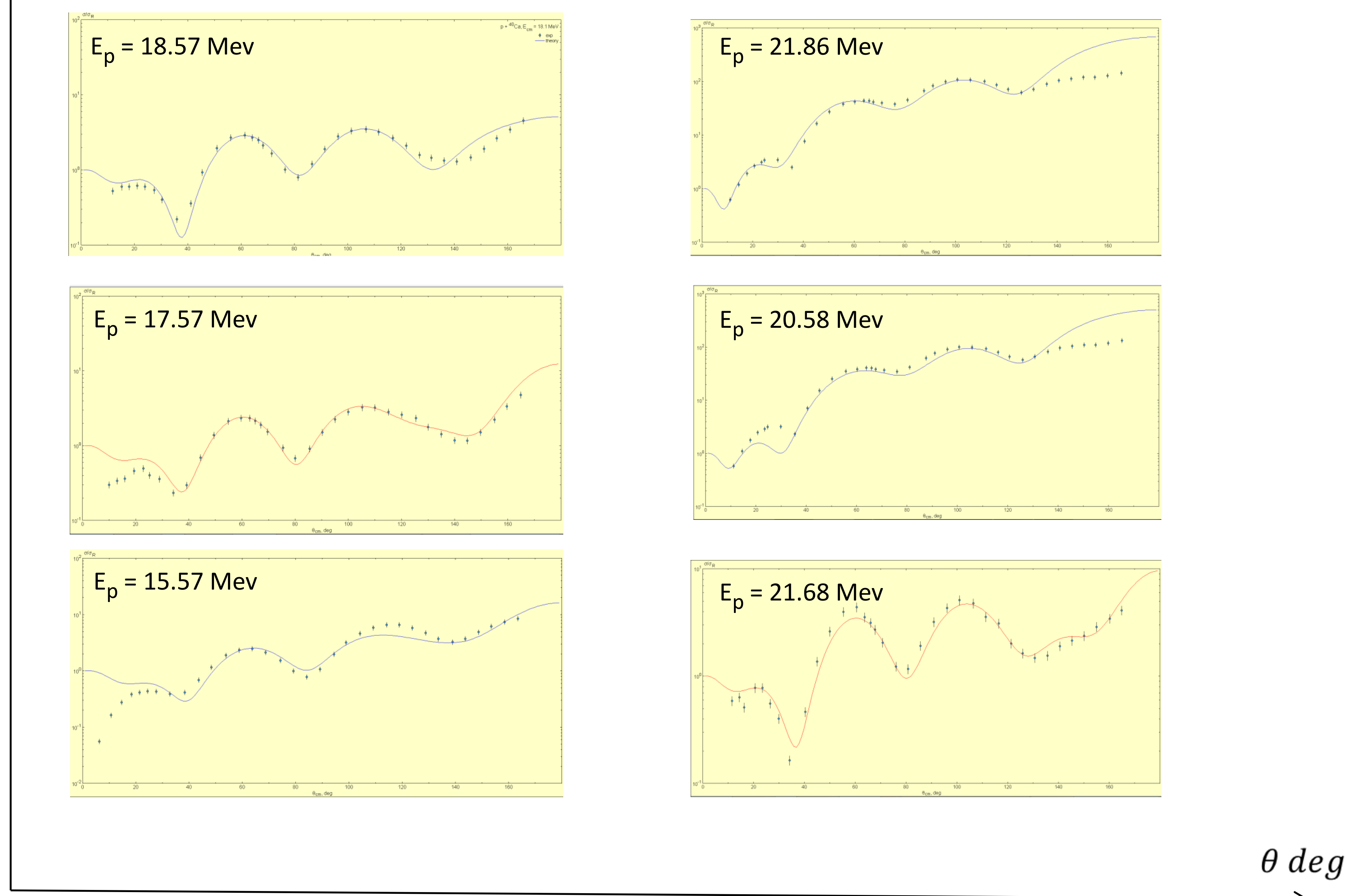
After fitting



Results


σ/σ_R
E_p = 9.86 Mev
E_p = 12.44 Mev
E_p = 10.37 Mev
E_p = 13.49 Mev
E_p = 11.42 Mev
E_p = 15.57 Mev
θ deg

Results


σ/σ_R
E_p = 18.57 Mev
E_p = 21.86 Mev
E_p = 17.57 Mev
E_p = 20.58 Mev
E_p = 15.57 Mev
E_p = 21.68 Mev
θ deg

Summary

- We studied basic aspects of the elastic scattering theory and the Optical Model.

- By choosing appropriate values of the parameters of the potential, we were able to construct a successful description of the cross section.

- Knowledge obtained in OM study is applicable for studying other non-elastic channels

Thanks **for your attention.**

Questions ?