JOINT INSTITUTE
FOR NUCLEAR RESEARCH

Computer simulation of tunneling characteristics of superconducting nanostructures

Presented By

Nayira Megahed El-Gammal

Assistant Lecture at Department of Physics-Faculty of Science
Menoufia University

Amr Hisham Khalaf Mahmoud

Assistant Lecture at Department of Basic Science-
Faculty of computer science and information
Ain Shams University

Laboratory of Theoretical Physics n.a. N.N. Bogolyubov, Joint Institute for Nuclear Research, Dubna, Russia

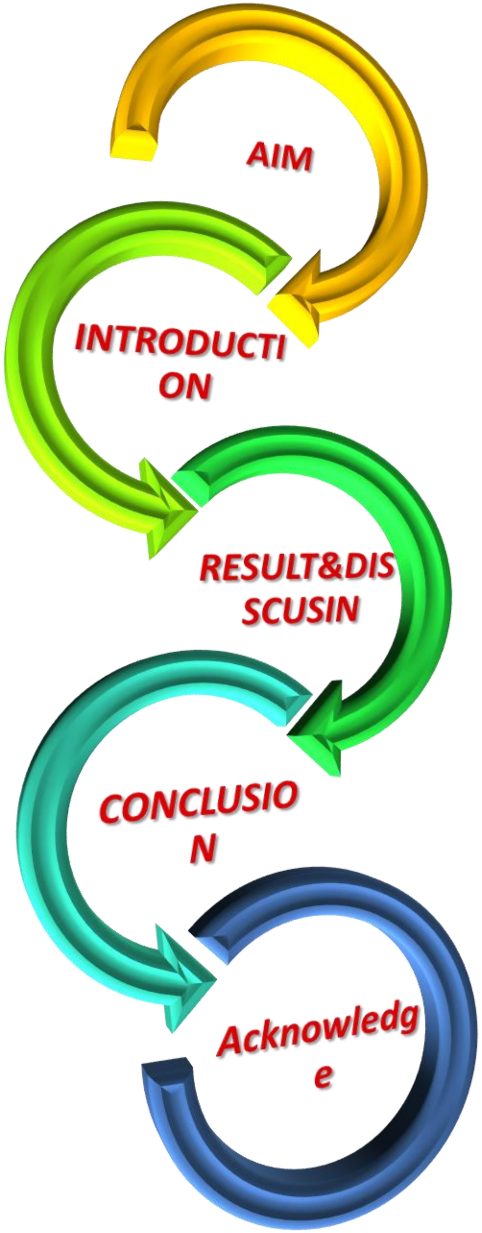
Under supervision of

Prof. Dr. Yu. Shukrinov

Dr. I. Rakhmonov

K. Kulikov

Outlines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CONCLUSIO N
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AIM OF WORK

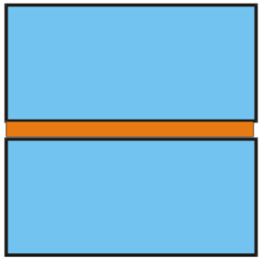
The aim of this work is to investigation of the shapiro step at nonequilibrium conditions.
(Nairaa)

investigate the effect of coupling between the superconducting current and magnetization in the superconductor/ferromagnet/superconductor Josephson junction under an applied circularly polarized magnetic field
(Amr)

Introduction

Josephson junction and the Josephson effect

A **Josephson junction** is a quantum mechanical device, which is made of two superconducting electrodes separated by a barrier (thin insulating tunnel barrier, normal metal, semiconductor, ferromagnet, etc.)



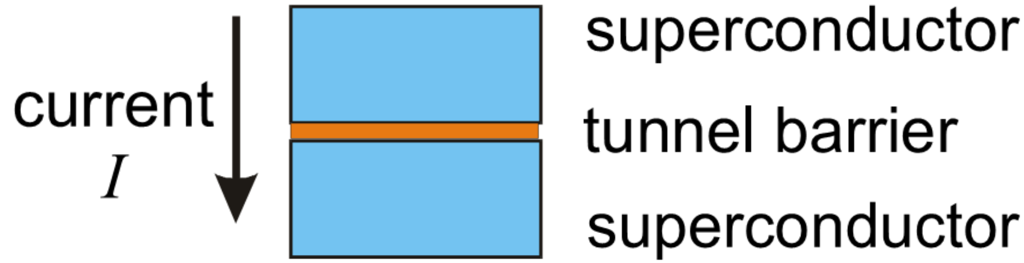
superconductor $\Psi_1 = |\Psi_1| \exp(i\theta_1)$

tunnel barrier

superconductor $\Psi_2 = |\Psi_2| \exp(i\theta_2)$

Phase difference: $\varphi = \theta_2 - \theta_1$

Josephson junction and the Josephson effect



dc Josephson effect: $I_s(\varphi) = I_c \sin\varphi \quad (1)$ $\quad I < I_C,\ V=0$

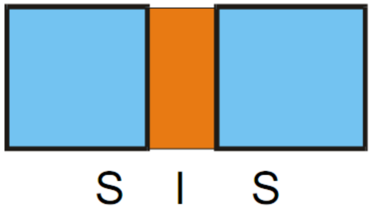
ac Josephson effect: $\frac{d\varphi}{dt} = \frac{2e}{\hbar} V \quad (2)$ $\quad I > I_C,\ V > 0$

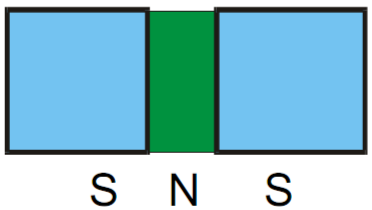
Obtained by B. Josephson in 1962

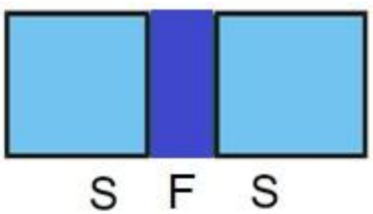
Josephson junction is a *quantum dc voltage - to - frequency converter*

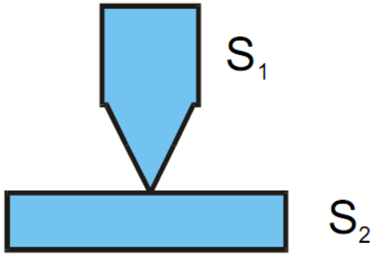
$$1\,\mu\text{V} \leftrightarrow 483.59767\ \text{MHz}$$

Types of Josephson junctions

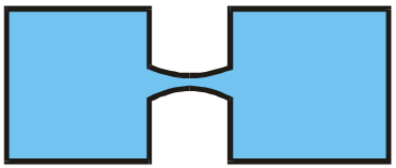






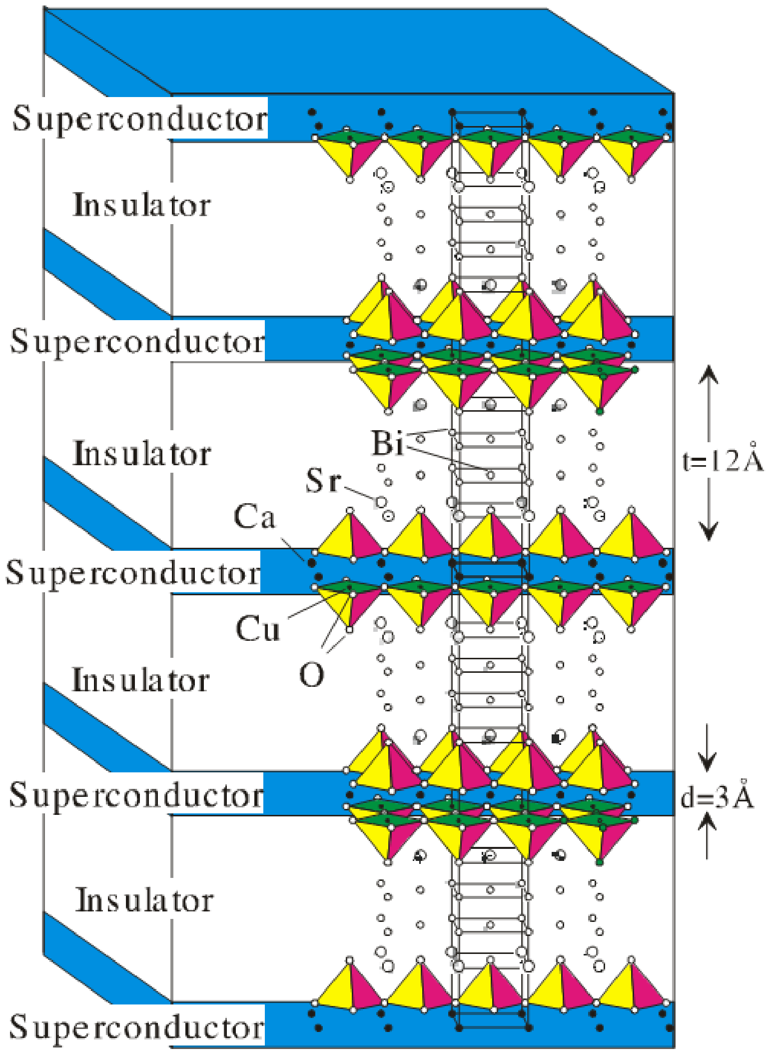


Point junction



Superconducting bridges

$Bi_2Sr_2CaCu_2O_{8+x}$



Natural stacks

RCSJ - Model

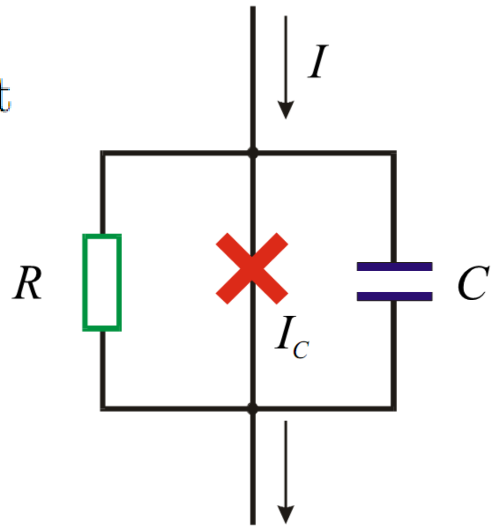
RCSJ ≡ "resistive-capacitive shunted junction"

The Josephson junction has a superconducting, capacitance and resistance properties. Therefore its an equivalent scheme has a following form

$$I_{qp} = \frac{V}{R} \text{–quasiparticle}$$

$$I_{disp} = C\frac{dV}{dt} \text{–displacement}$$

$$I_s = I_c \sin\varphi \text{–supercomducting}$$



$$I = C\frac{dV}{dt} + I_c \sin\varphi + \frac{V}{R}$$

Summation of current

RCSJ - Model

Using AC Josephson relation and summation of current we can write the system of equations in normalized units to describe the electromagnetic properties of the Josephson junction

$$\begin{cases} \dfrac{\partial \varphi}{\partial t} = V \\ \dfrac{dV}{dt} = I - \sin\varphi - \beta \dfrac{\partial \varphi}{\partial t} \end{cases}$$

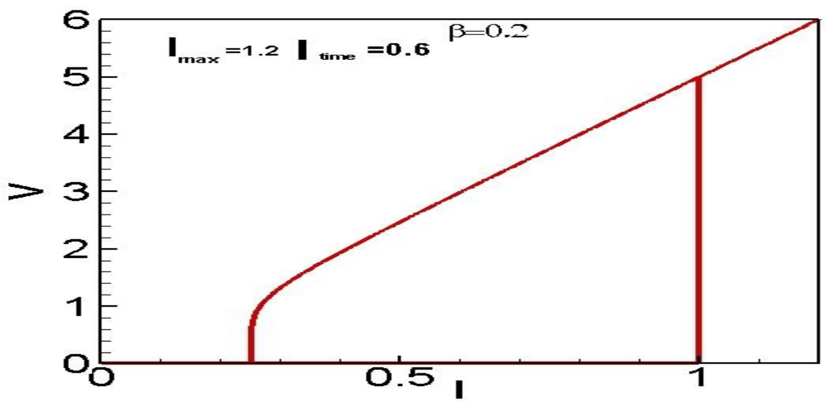
Here V is Voltage and its normalized to $V_0 = \dfrac{\hbar \omega_p}{2e}$;

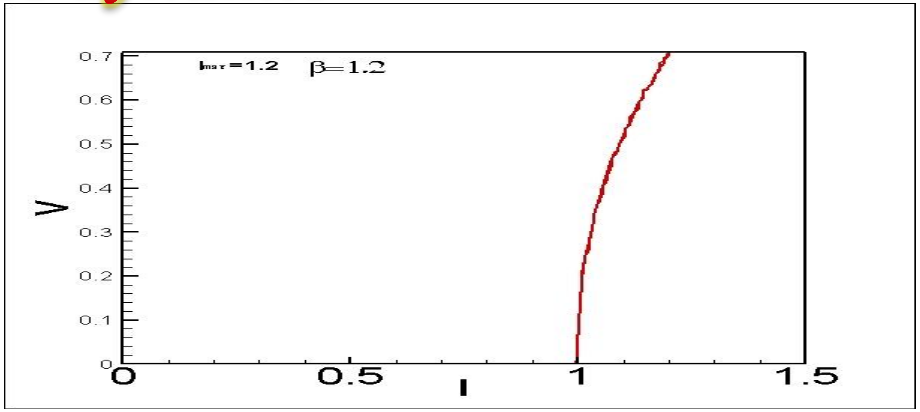
$\omega_p = \sqrt{\dfrac{2eI_c}{C\hbar}}$ Josephson plasma frequency

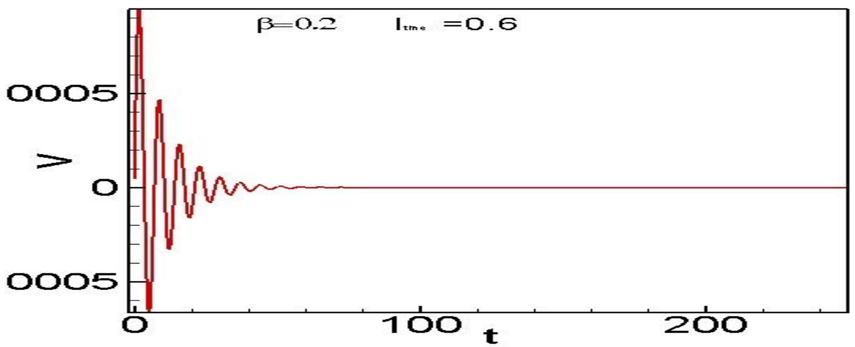
$\beta = \dfrac{1}{R}\sqrt{\dfrac{\hbar}{2eI_c C}}$ dissipation parameter

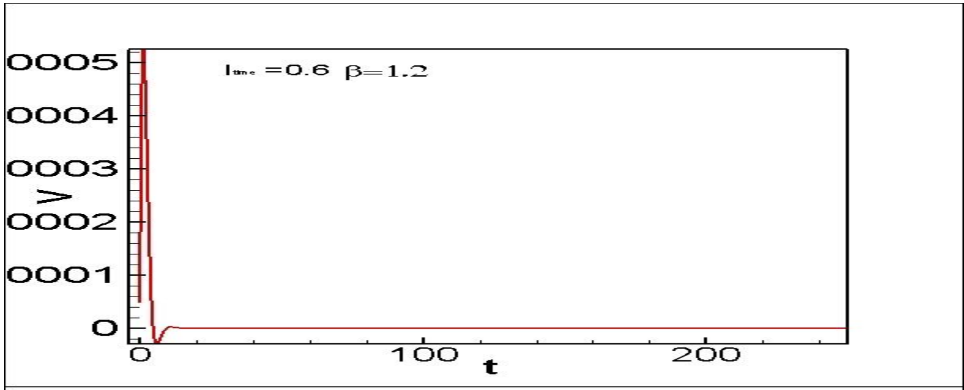
Current I is normalized to the citical current I_c

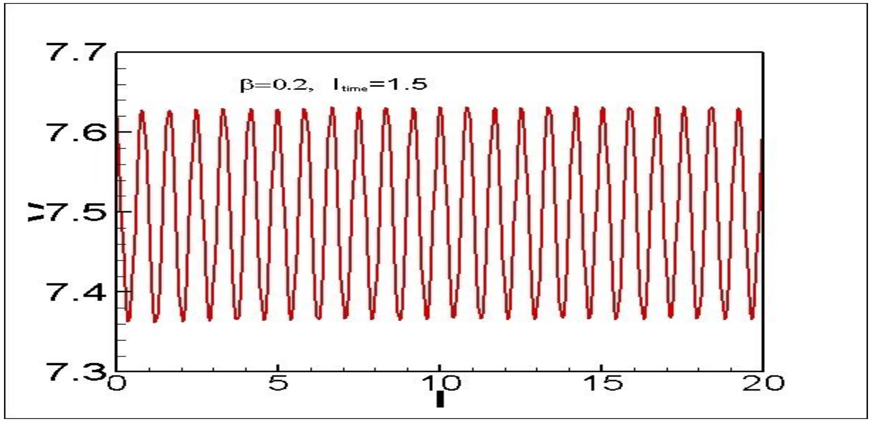
CVC and phase dynamic

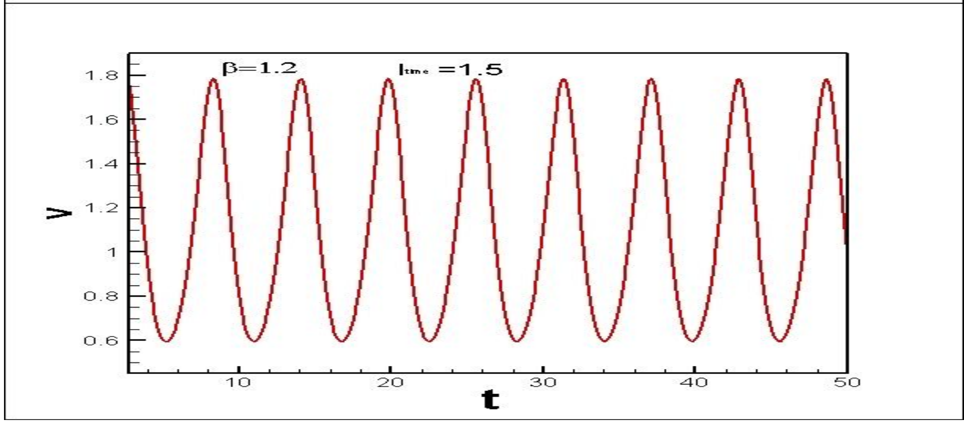




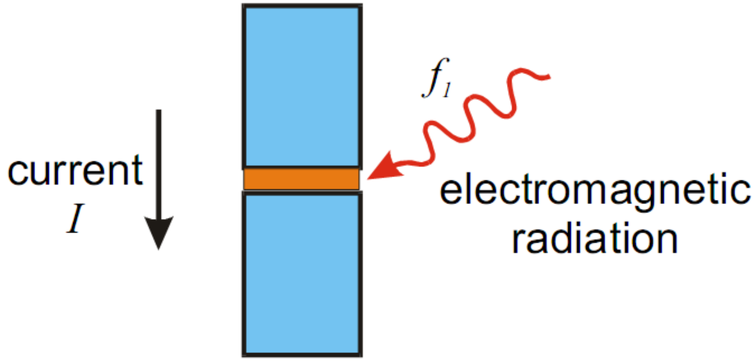








The influence of external radiation and the Shapiro step

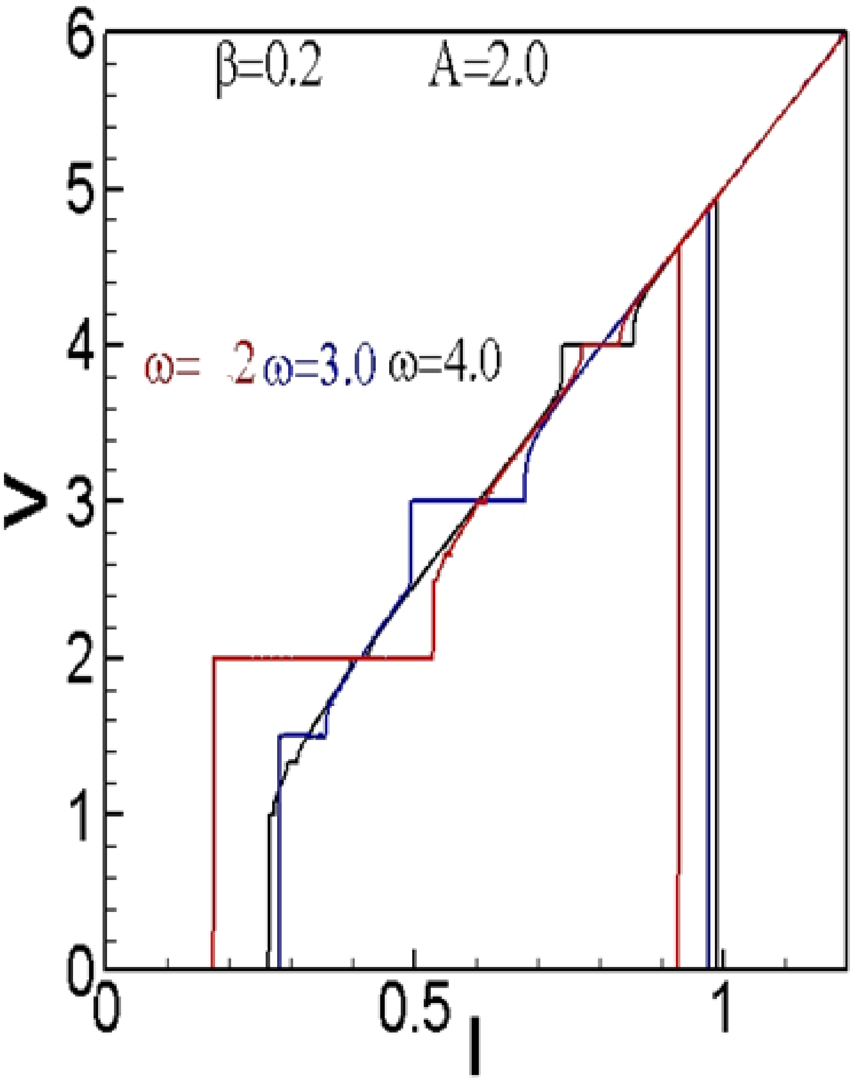


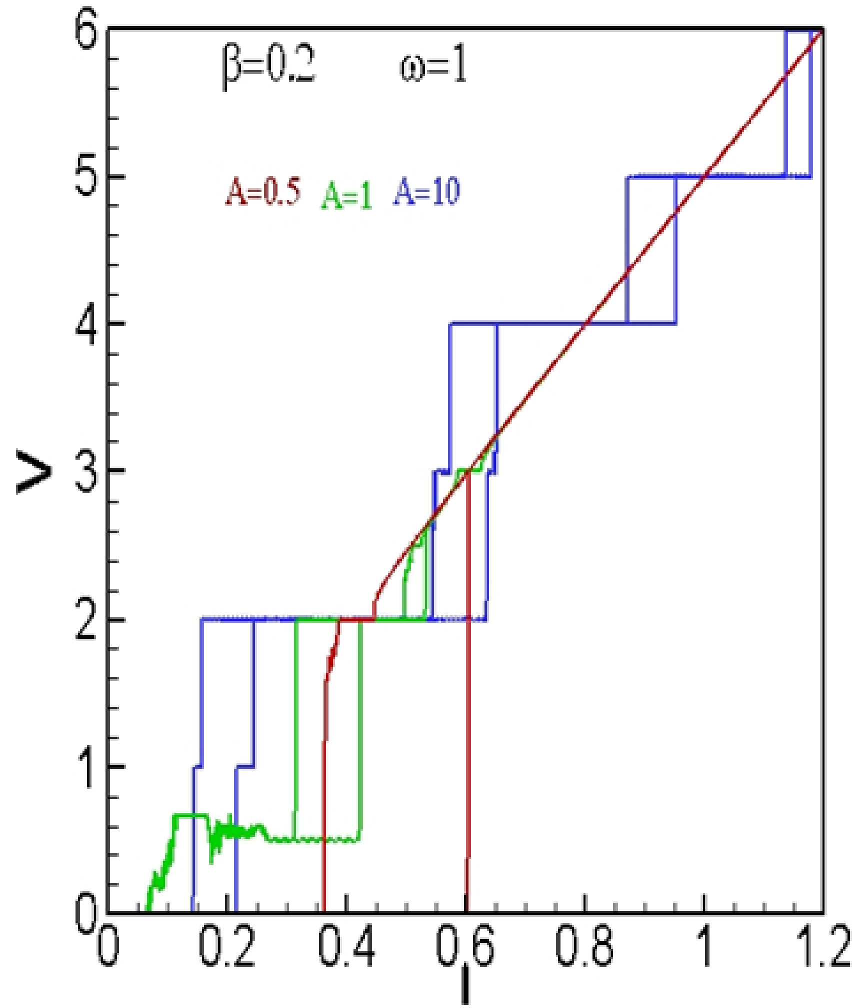
$$\begin{cases} \dfrac{\partial \varphi}{\partial t} = V \\ \dfrac{dV}{dt} = I - \sin\varphi - \beta \dfrac{\partial \varphi}{\partial t} + A \sin \omega t \end{cases}$$

$$I_s = I_c \sum_{n=0}^{\infty} (-1)^n J_n\left(\frac{V_1}{\Phi_0 f_1}\right) \sin\left[\varphi_0 + \frac{2\pi}{\Phi_0} V_0 t - 2\pi n f_1 t\right]$$

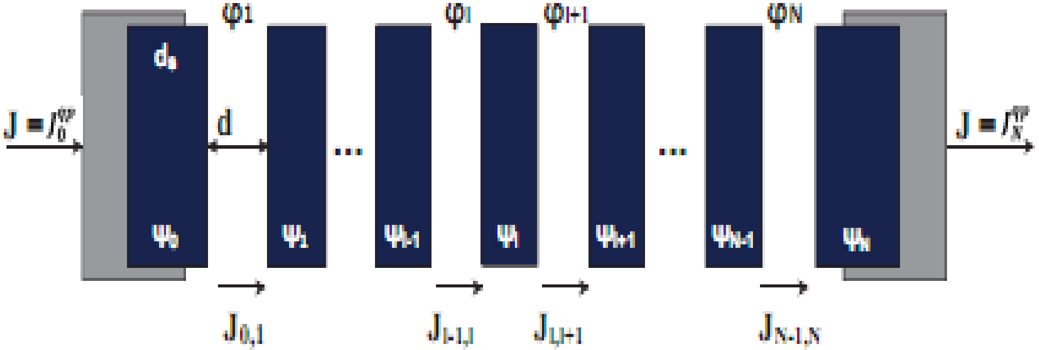
$$\Delta I = 2|J_n(f)|, \qquad f = \frac{A}{\omega} \frac{1}{\sqrt{\beta^2 + \omega^2}}$$

Shapiro step on CVC





Shapiro step at nonequilibrium conditions



Layered system of N+1 superconducting layers forms a stack of Josephson junctions.

Generalized Josephson relation

$$\frac{\mathrm{d}\varphi_l(t)}{\mathrm{d}t} = \frac{2e}{\hbar}\Big(V_l(t) + \Phi_l(t) - \Phi_{l-1}(t)\Big)$$

The total current density through each S-layer is given as a sum of displacement, superconducting, quasiparticle and diffusion terms:

$$J_l = C\frac{\mathrm{d}V_l}{\mathrm{d}t} + J_c \sin\varphi_l + \frac{\hbar}{2eR}\dot{\varphi}_l + \frac{\Psi_{l-1} - \Psi_l}{R},$$

kinetic equations for Ψ_l

$$\frac{\partial \Psi_l}{\partial t} = \frac{4\pi r_D^2}{d_s^i}\left(J_l^{qp} - J_{l-1}^{qp}\right) - \frac{\Psi_i}{\tau_{qp}}$$

Yu. M. Shukrinov1, M. Nashaat1,K.V.Kulikov1,R.Dawood1,H.ElSamman and Th. M. El Sherbini., EPL, 115 (2016) 20003

In the normalized form the system of equations are:

$$\dot{v}_l = \Big[I - \sin\varphi_l - \beta\dot{\varphi}_l + A\sin\omega\tau + I_{noise} + \psi_l - \psi_{l-1} \Big],$$

$$\dot{\varphi}_1 = v_1 - \alpha(v_2 - (1+\gamma)v_1) + \frac{\psi_1 - \psi_0}{\beta},$$

$$\dot{\varphi}_l = (1+2\alpha)v_l - \alpha(v_{l-1} + v_{l+1}) + \frac{\psi_l - \psi_{l-1}}{\beta},$$

$$\dot{\varphi}_N = v_N - \alpha(v_{N-1} - (1+\gamma)v_N) + \frac{\psi_N - \psi_{N-1}}{\beta},$$

$$\zeta_0\dot{\psi}_0 = \eta_0(I - \beta\dot{\varphi}_{0,1} + \psi_1 - \psi_0) - \psi_0,$$

$$\zeta_l\dot{\psi}_l = \eta_l(\beta[\dot{\varphi}_{l-1,l} - \dot{\varphi}_{l,l+1}] + \psi_{1-1} + \psi_{l+1} - 2\psi_l) - \psi_l,$$

$$\zeta_N\dot{\psi}_N = \eta_N(-I + \beta\dot{\varphi}_{N-1,N} + \psi_{N-1} - \psi_N) - \psi_N,$$

Results

In the normalized form the system of equations are:

dissipation parameter

$$\beta = \frac{\hbar\omega_p}{2eRI_c}$$

quasiparticle relaxation time

$$\tau = \omega_p t$$

plasma frequency

$$\omega_p = \sqrt{\frac{2eJ_c}{\hbar C}}$$

coupling parameter

$$\alpha = \epsilon\epsilon_0/2e^2N(0)d \quad \text{i}$$

nonequilibrium parameter

$$\eta_l = \frac{4\pi r_D^2 \tau_{qp}}{d_s^l R}$$

normalized quasiparticle relaxation time

$$\zeta_l = \omega_p \tau_{qp}$$

$$\dot{v}_l = \left[I - \sin\varphi_l - \beta\dot{\varphi}_l + A\sin\omega\tau + I_{noise} + \psi_l - \psi_{l-1}\right],$$

$$\dot{\varphi}_1 = v_1 - \alpha(v_2 - (1+\gamma)v_1) + \frac{\psi_1 - \psi_0}{\beta},$$

$$\dot{\varphi}_l = (1+2\alpha)v_l - \alpha(v_{l-1} + v_{l+1}) + \frac{\psi_l - \psi_{l-1}}{\beta},$$

$$\dot{\varphi}_N = v_N - \alpha(v_{N-1} - (1+\gamma)v_N) + \frac{\psi_N - \psi_{N-1}}{\beta},$$

$$\zeta_0\dot{\psi}_0 = \eta_0\left(I - \beta\dot{\varphi}_{0,1} + \psi_1 - \psi_0\right) - \psi_0,$$

$$\zeta_l\dot{\psi}_l = \eta_l\left(\beta[\dot{\varphi}_{l-1,l} - \dot{\varphi}_{l,l+1}] + \psi_{1-1} + \psi_{l+1} - 2\psi_l\right) - \psi_l,$$

$$\zeta_N\dot{\psi}_N = \eta_N\left(-I + \beta\dot{\varphi}_{N-1,N} + \psi_{N-1} - \psi_N\right) - \psi_N,$$

charge imbalance potential

$$\Psi_n = \frac{\tau_{qp}}{2e^2N(0)}\left(J_{l-1}^{qp} - J_l^{qp}\right)$$

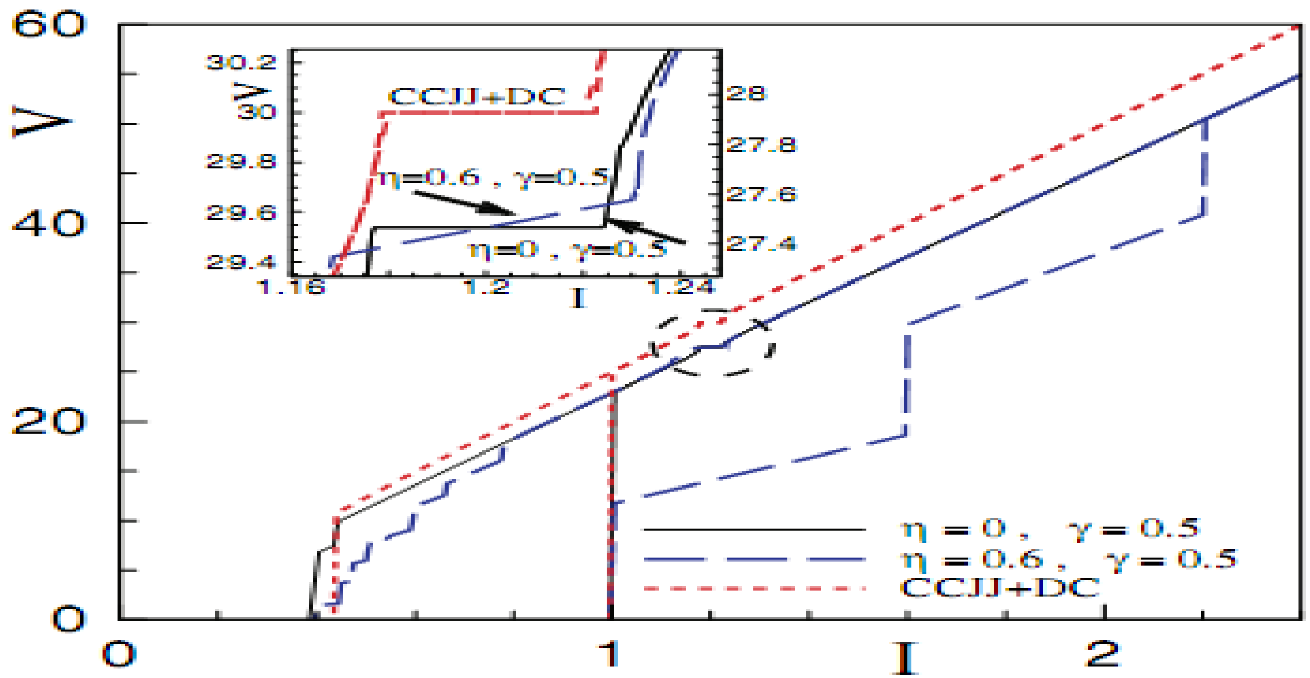
nonperiodic boundary conditions

$$\gamma = \frac{d_s}{d_s^0} = \frac{d_s}{d_s^n}.$$

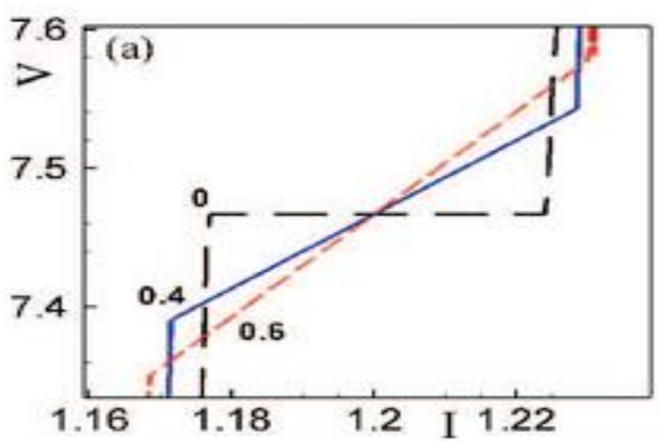
effect of external radiation

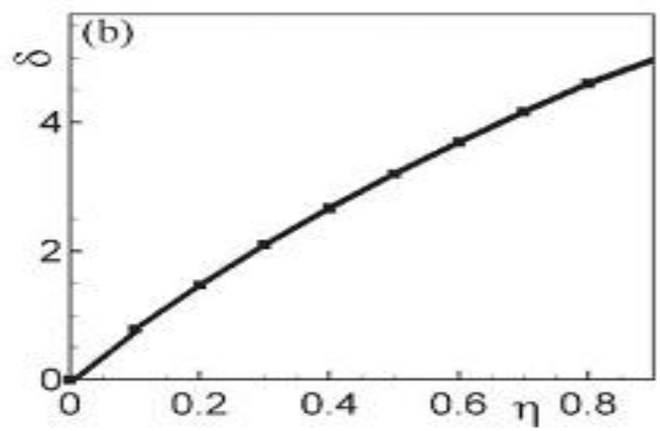
$$A\sin\omega\tau$$

Slope and shift of shapiro step



N=5,
β=0.2,
ω=6,
A=1.6,
ζ=0.1,
α=0.5,
γ=0.5.





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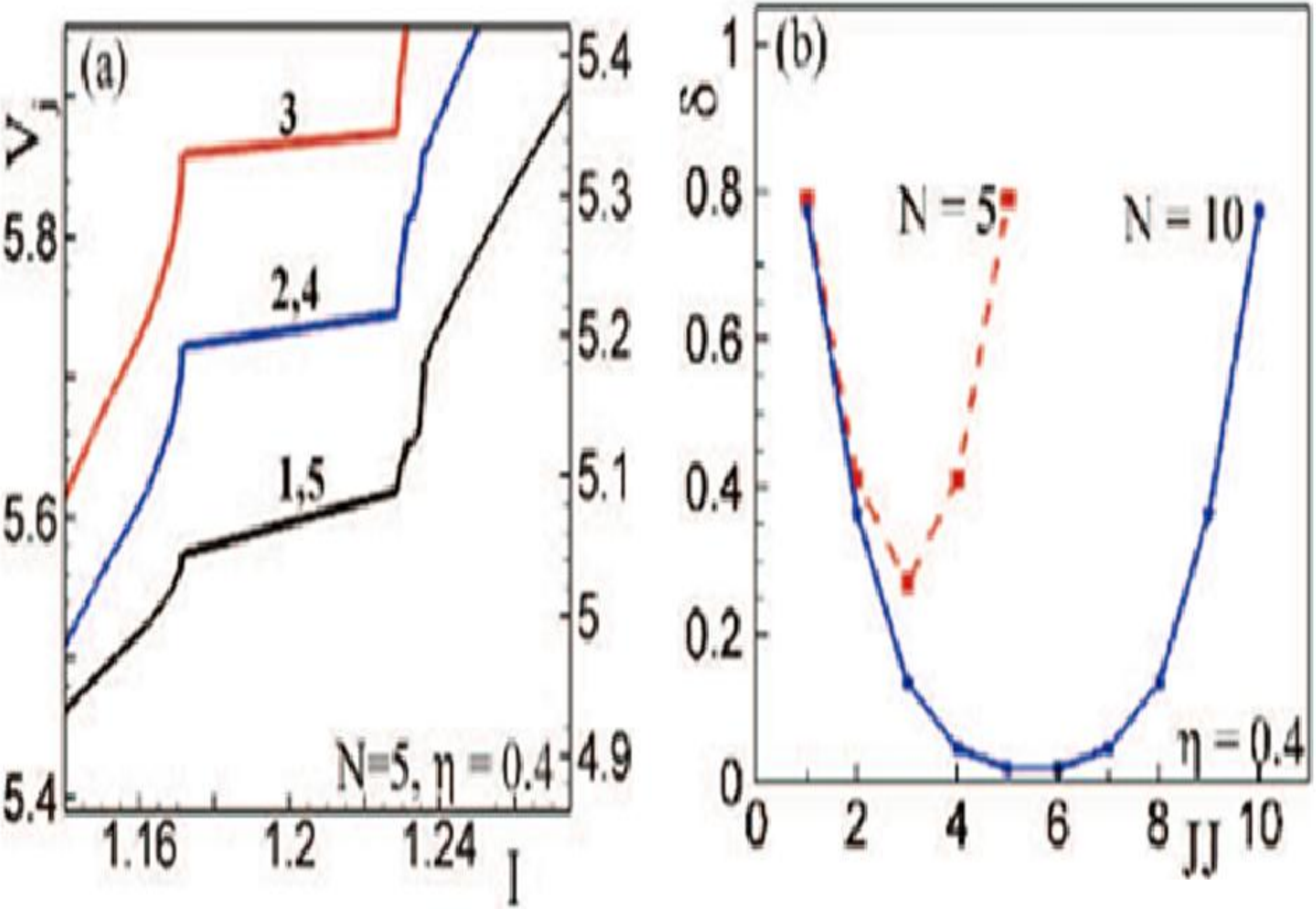

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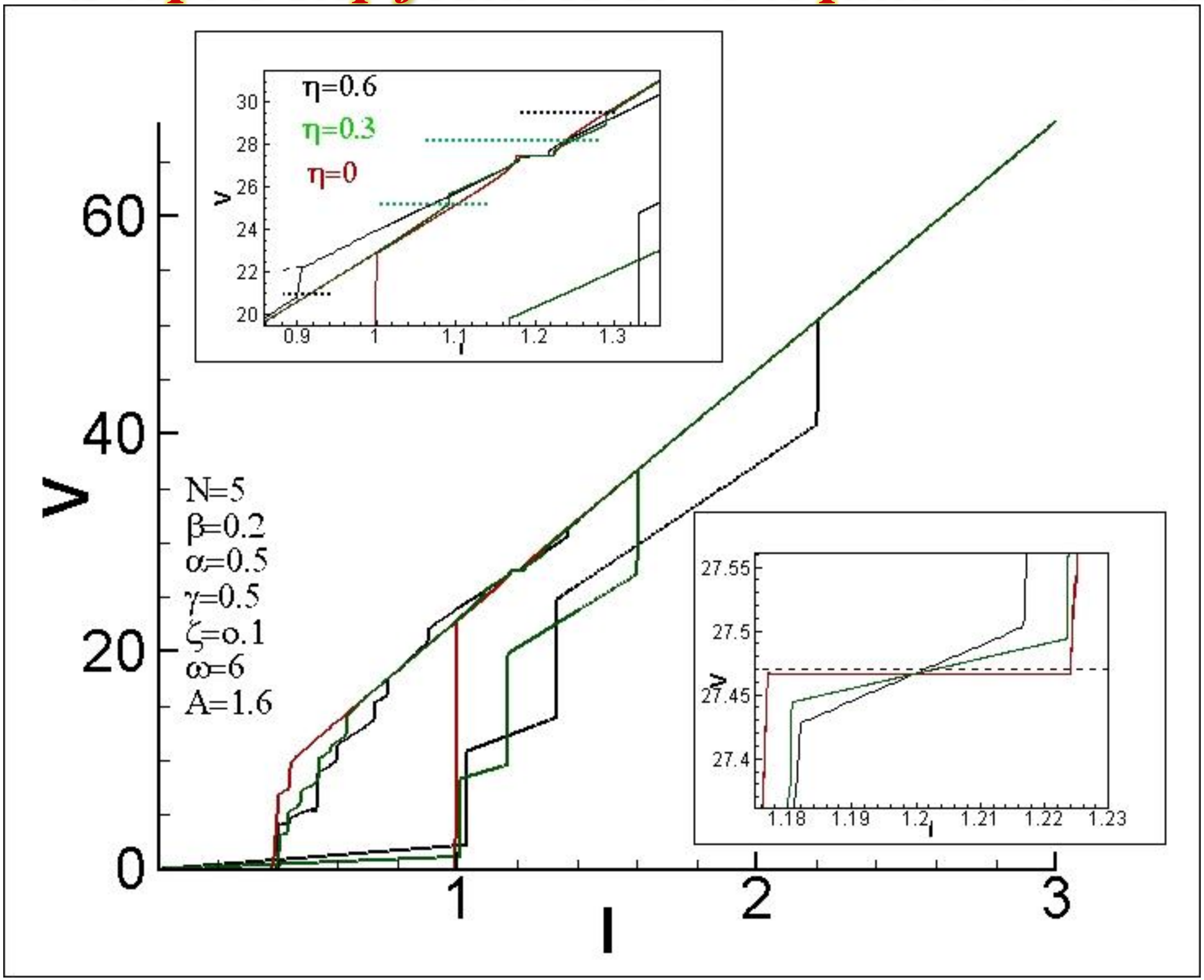
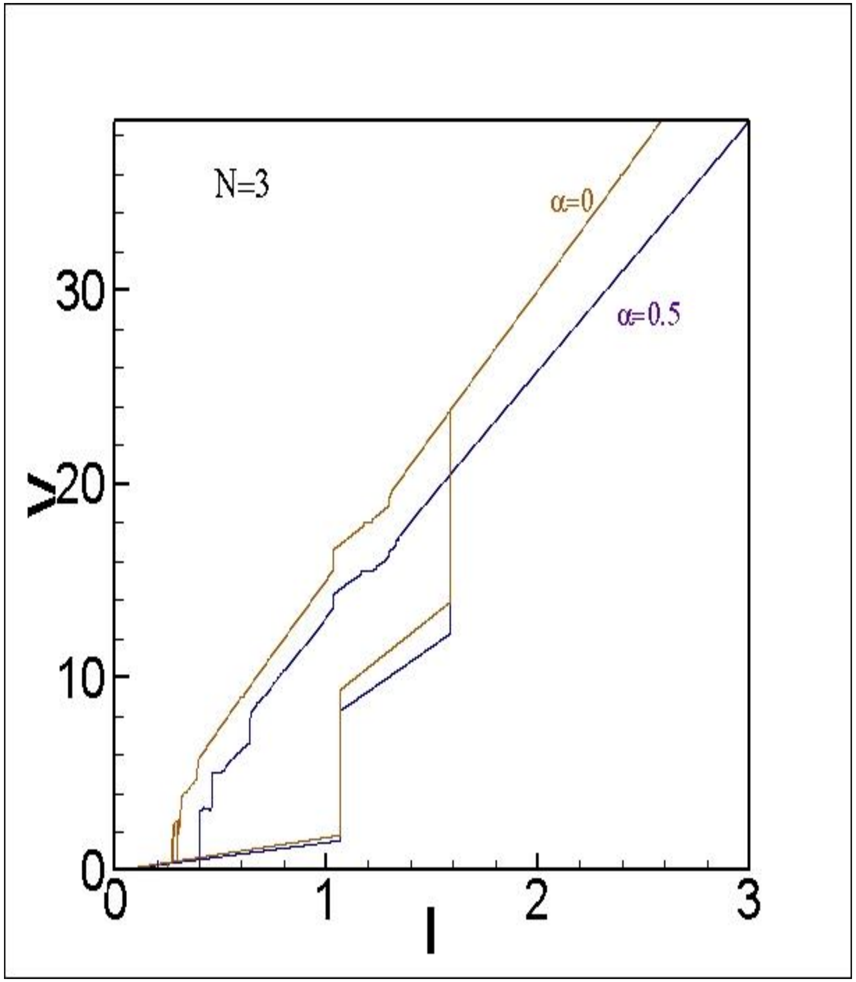
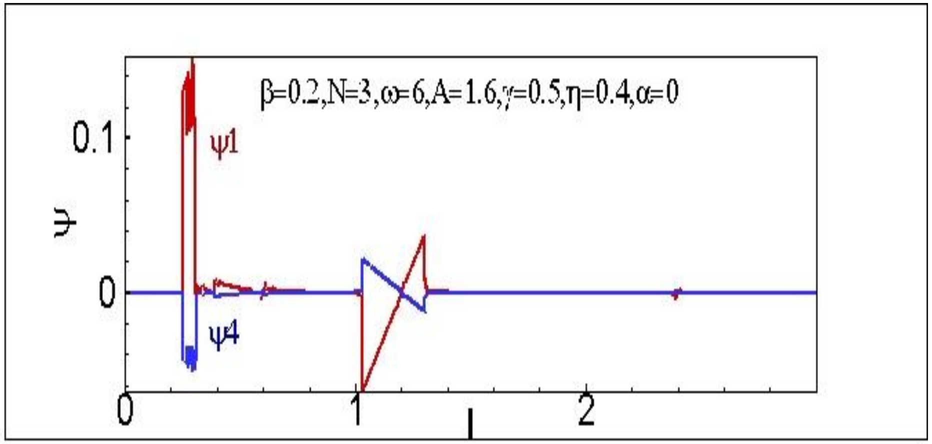
Shapiro step features at nonequilibrium conditions

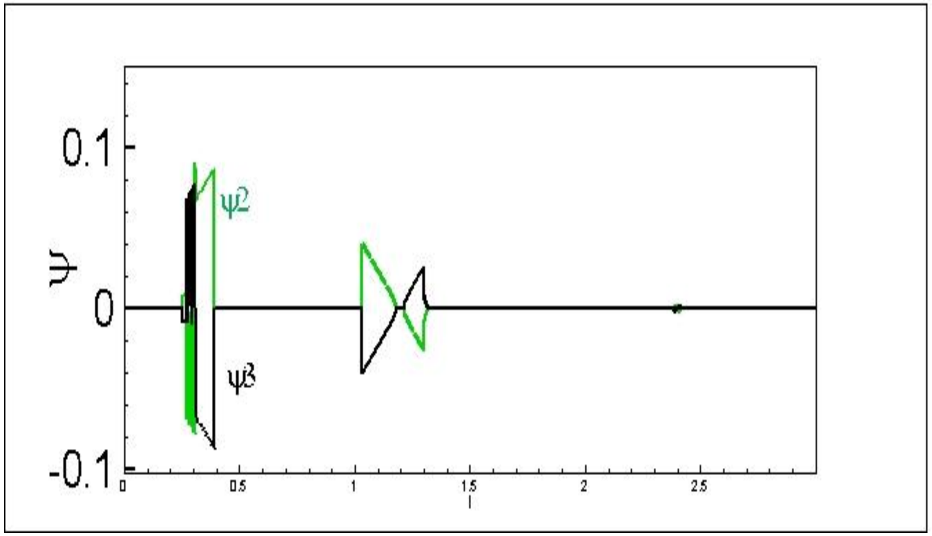


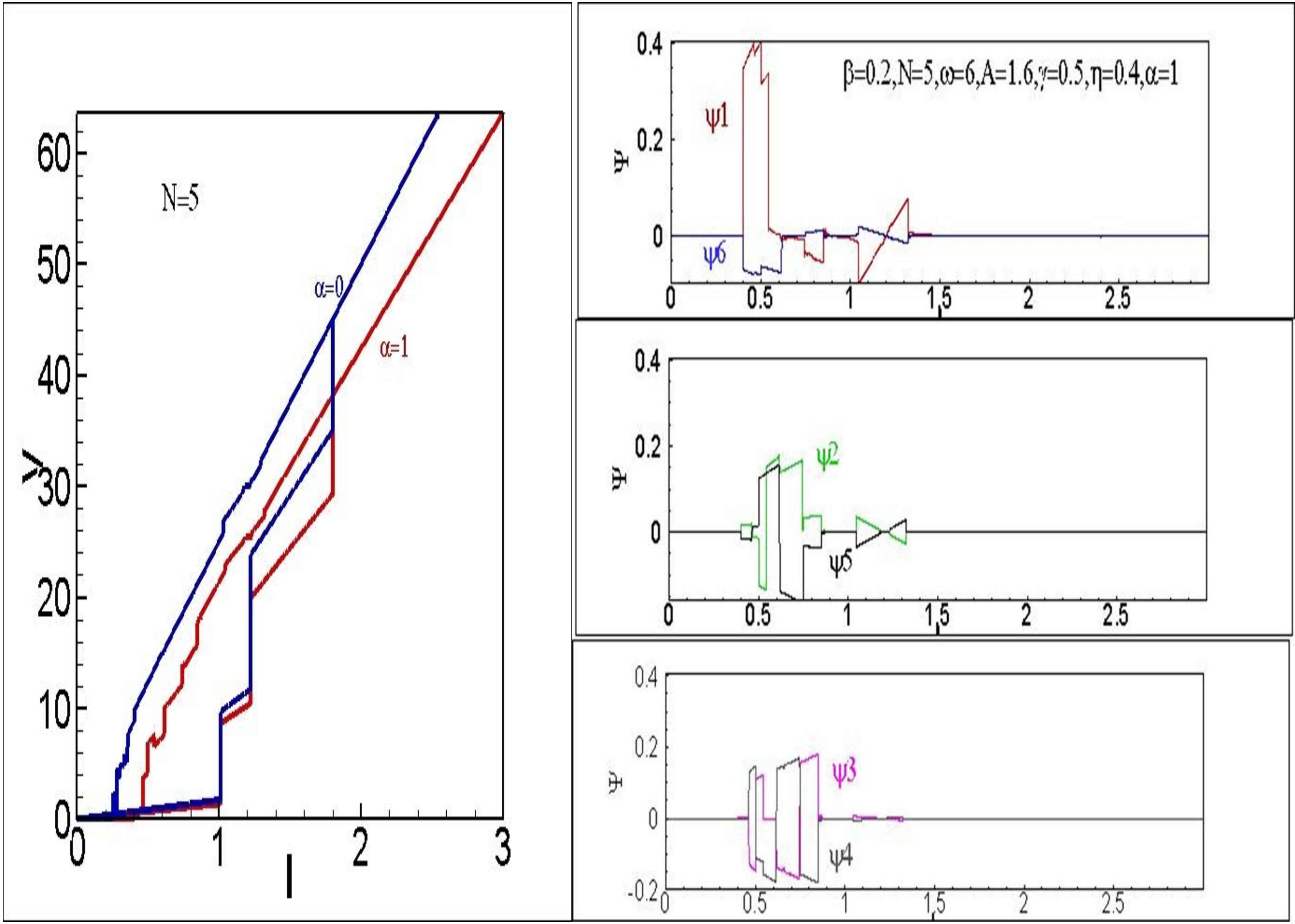
N=5,
β=0.2,
ω=6,
A=1.6,
ζ=0.1,
α=0.5,
γ=0.5.

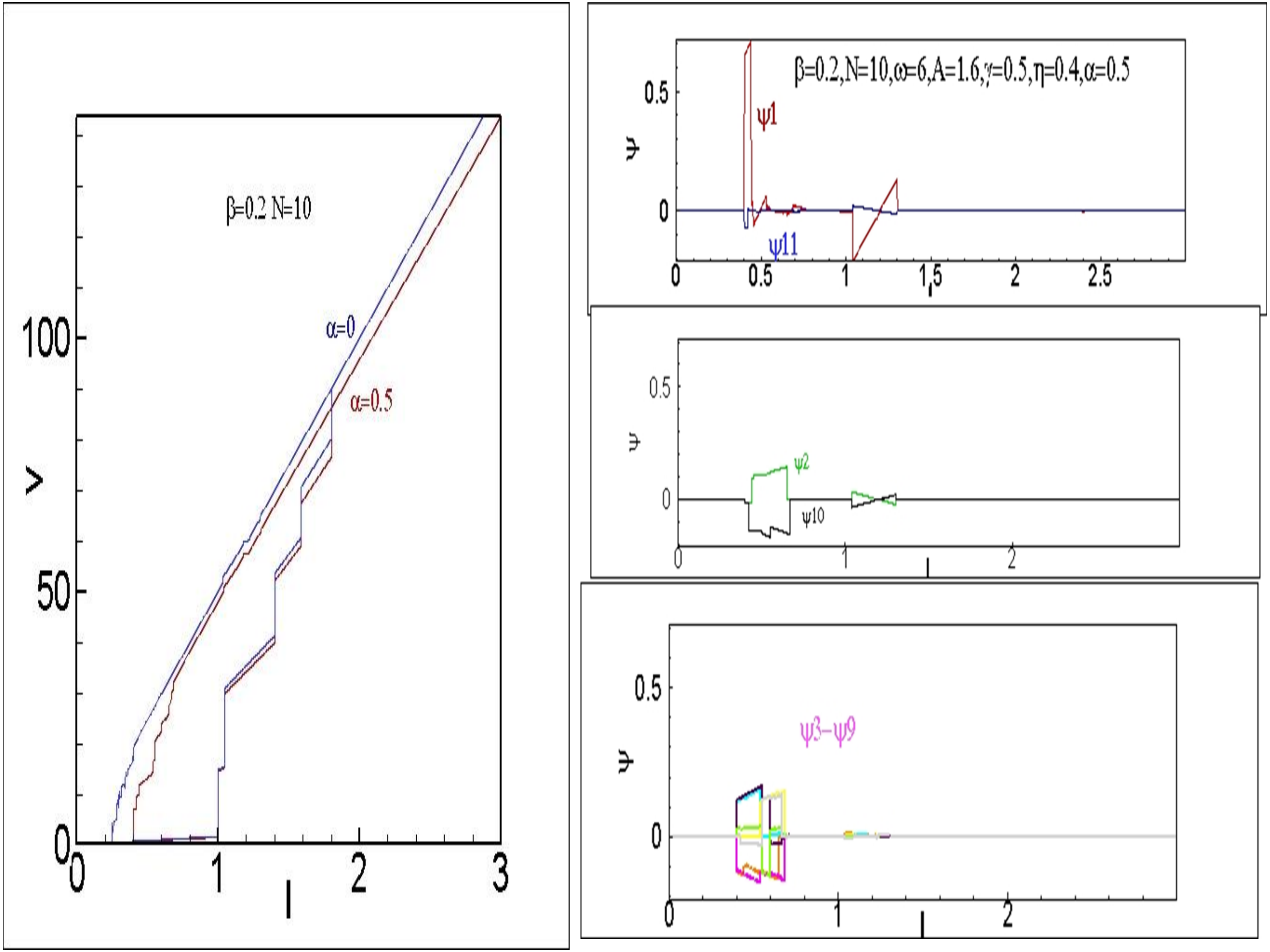
Nonequilibrium potential in the system of coupled Josephson junction CJJ








N=5
α=0
α=1
V
I
ψ1
ψ6
β=0.2,N=5,ω=6,A=1.6,γ=0.5,η=0.4,α=1
Ψ
ψ2
ψ5
ψ3
ψ4


β=0.2 N=10
α=0
α=0.5
V
I
100
50
0
1
2
3
β=0.2,N=10,ω=6,A=1.6,γ=0.5,η=0.4,α=0.5
ψ1
ψ11
ψ
0.5
1.5
2.5
ψ2
ψ10
ψ3−ψ9

conclusion

- Studying J.J effect and how to caculate I-V dynamic of the system of Josephson effect s.
- Studying the Charge imbalance on a stack of Josephson junctions and on Shapiro step
- New feature appears in I-V characteristics (Jumps) due to take on account AC.

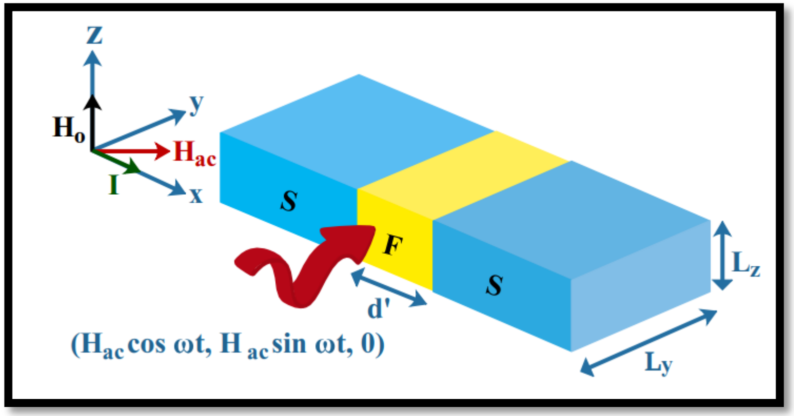
Devil's staircases in the IV characteristics of superconductor/ferromagnet/superconductor Josephson junctions

M. Nashaat,[1,2] A. E. Botha,[3] and Yu. M. Shukrinov[2,3,4,*]
[1]*Department of Physics, Cairo University, Cairo, 12613, Egypt*
[2]*BLTP, JINR, Dubna, Moscow Region, 141980, Russia*
[3]*Department of Physics, University of South Africa, Science Campus, Private Bag X6, Florida Park 1710, South Africa*
[4]*Dubna State University, 141982 Dubna, Russian Federation*

Aim

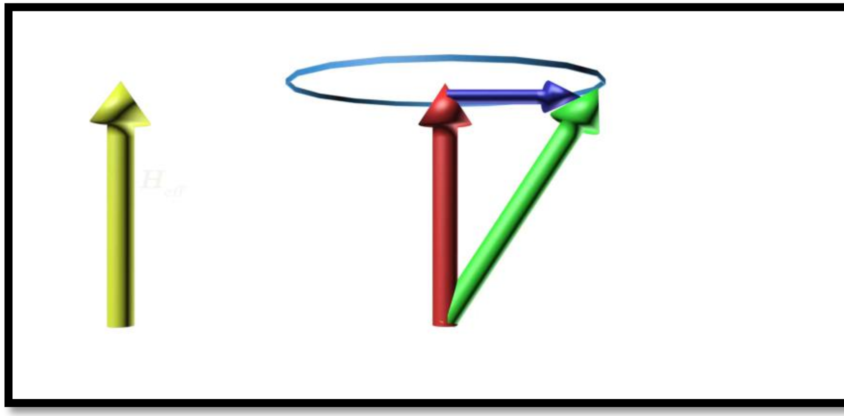
In this paper we investigate the effect of coupling between the superconducting current and magnetization in the superconductor/ferromagnet/superconductor Josephson junction under an applied circularly polarized magnetic field

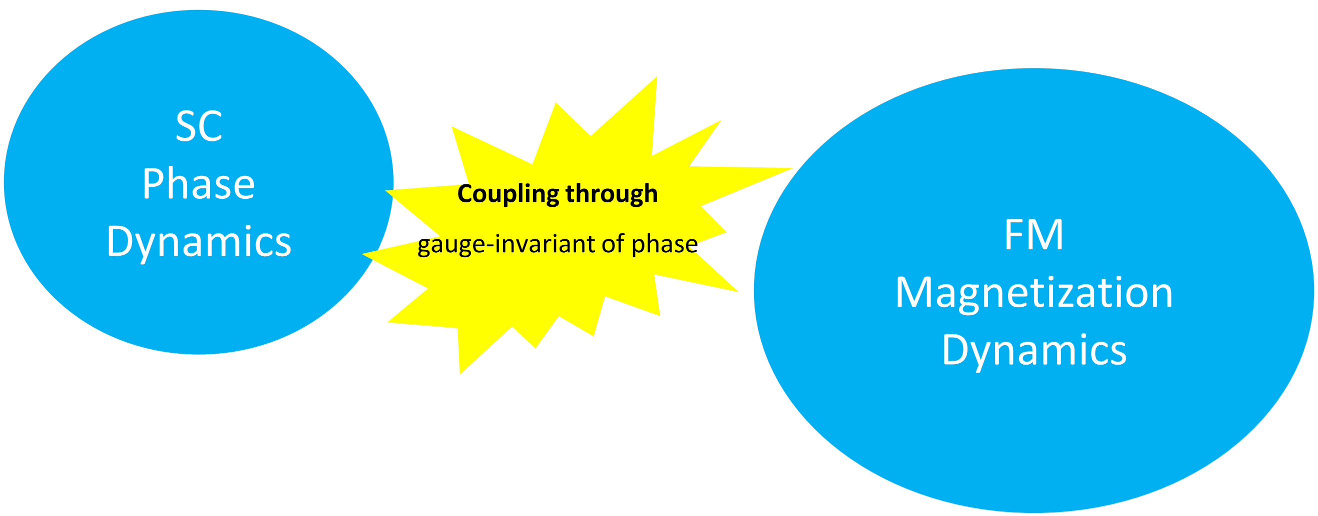
SFS under Magnetic field



SFS Josephson junction under circularly polarized magnetic field in xy-plane with amplitude H_{ac} and frequency ω.

Spin wave Excitation





Extended RCSJ Model:

The gauge-invariant phase difference is given by[1]:

$$\Theta(y,z,t) = \theta(t) - \frac{8\pi^2\, dM_z(t)}{\phi_0} y + \frac{8\pi^2\, dM_y(t)}{\phi_0} z$$

Θ : gauege invariant phase difference

$\phi_0 = \frac{h}{2e}$: magnetic flux quantum

The RCSJ equation reads

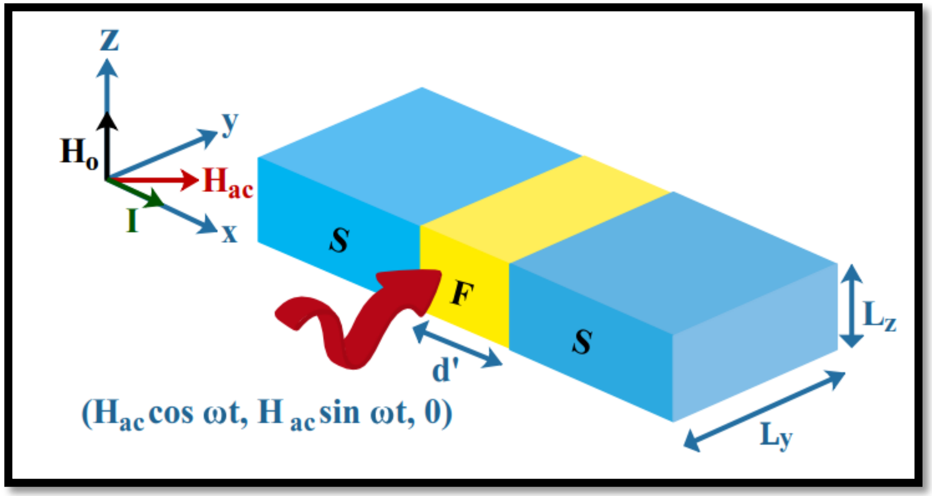
I/I_c^0 : electric current

$$I/I_c^0 = \frac{\phi_0^2 \sin\theta \sin\left(\frac{4\pi^2\, dMz(t)Ly}{\phi_0}\right)\sin\left(\frac{4\pi^2\, dMy(t)Lz}{\phi_0}\right)}{16\,\pi^4 d^2 L_z L_y M_z(t) M_y(t)}$$

$$+ \frac{\phi_0}{2\pi R I_c^0}\frac{d\theta(y,z,t)}{dt} + c\,\frac{\phi_0}{2\pi I_c^0}\frac{d^2\theta(y,z,t)}{dt^2}$$

$$I/I_c^0 = \frac{\sin\theta(\tau)\sin\left(\frac{\pi\,\phi z(\tau)}{\phi_0}\right)\sin\left(\frac{\pi\,\phi y(\tau)}{\phi_0}\right)}{\left(\frac{\pi\,\phi y(\tau)}{\phi_0}\right)\left(\frac{\pi\,\phi z(\tau)}{\phi_0}\right)}$$

$$+ \frac{d\theta(\tau)}{d\tau} + \beta_c \frac{d^2\theta(\tau)}{d\tau^2}$$



[1]S Hikino, M Mori, S Takahashi, and S Maekawa. Superconductor Science and Technology, 24(2):024008, 2011

$\tau = t\,\omega_c$:Normalized time

$\omega_c = 2\pi\, I_c^0 R/\Phi_0$:Characteristics frequency

β_c :McCumber (fixed parameter)

C :Junction Capacitor

M :Magentization

Extended RCSJ Model:

Landau-Lifshitz-Gilbert (LLG) equation

$$(1+\alpha^2)\frac{dM}{dt} = -(\gamma M \times H_e + \frac{\gamma\alpha}{|M|}[M \times (M \times H_e)])$$

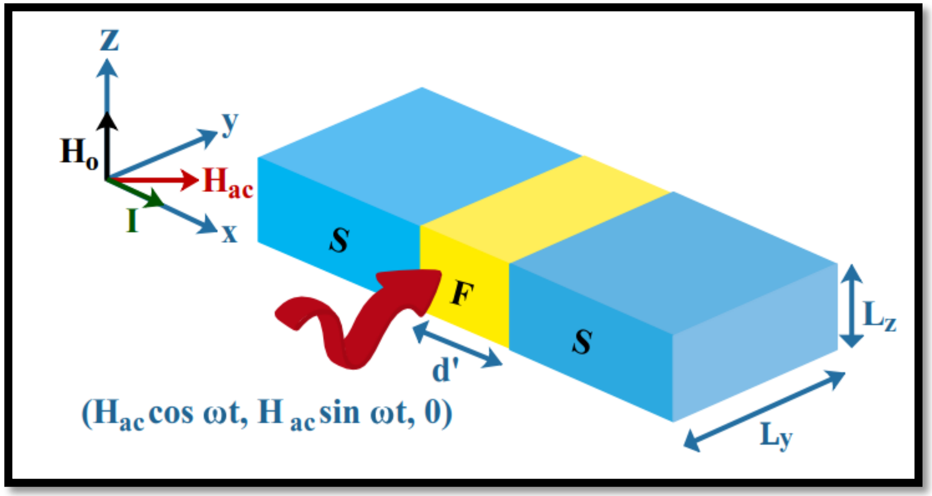
H_e : effective field
γ :gyromagnetic ratio (fixed)
α : Gilbert damping (fixed)

The effective field is calculated from

$$H_e = -\frac{1}{v}\frac{\partial E}{\partial M}$$

$$E = E_s + E_M + E_{ac}$$

$$E_s = E_j[1 - \cos(\theta(y,z,t))]$$
$$E_M = -H_o M_z$$
$$E_{ac} = -vM_x H_{ac}\cos\omega t - vM_y H_{ac}\sin\omega t$$



[1]S Hikino, M Mori, S Takahashi, and S Maekawa. Superconductor Science and Technology, 24(2):024008, 2011

In dimensionless Form:

$m = \frac{M}{|M|}$ **Normalized magnetization**

$h_e = \frac{He}{Ho}$ **Normalized effective magnetic field**

$\varepsilon_J = \frac{E_j}{V\,|M|\,H_0}$ **Normalized Josephson Energy**

$\beta c = \omega_c RC$ **McCumber (fixed parameter)**

$h_{ac} = \frac{H_{ac}}{H_o}$ **Normalized polarized magnetic field**

$\Omega = \frac{\omega}{\omega_c}$ **Normalized external Frequency**

$t = \tau\omega_c$ **Normalized Time**

$\omega_c = 2\pi\, I_c^0\, R/\Phi_0$ **Characteristic Frequency**

$H_0 = \frac{\Omega_0}{\gamma}$ **Applied Uniform Field in Z direction**

$\Omega_0 = \frac{\omega_0}{\omega_c}$ **Normalized internal Frequency**

$\phi_{sy} = \frac{4\pi^2 L_y\, d|M|}{\phi_0}$ **Phase difference In Y direction**

$\phi_{sz} = \frac{4\pi^2 L_z\, d|M|}{\phi_0}$ **Phase difference In Z direction**

In dimensionless Form:

$$m = \frac{\mathbf{M}}{|M|}, \quad h_e = \frac{H_e}{H_o}, \quad \epsilon_J = \frac{E_J}{\mathrm{v}\,|M|H_o}, \quad t = \tau\omega_c, \quad \omega_c = 2\pi I_c^0 R/\Phi_o, \quad H_o = \Omega_o/\gamma$$

$$\beta_c = \omega_c RC, \quad h_{ac} = \frac{H_{ac}}{H_o}, \quad \Omega = \omega/\omega_c, \quad \Omega_o = \omega_o/\omega_c, \quad \phi_{sy} = \frac{4\pi^2 L_y d|M|}{\Phi_o}, \quad \phi_{sz} = \frac{4\pi^2 l_z d|M|}{\Phi_o}$$

❑ Exact Numerical Scheme

$$\frac{d\mathbf{m}}{dt} = -\frac{\Omega_0}{(1+\alpha^2)}\left(\mathbf{m}\times\mathbf{h}_e + \alpha\left[\mathbf{m}\times(\mathbf{m}\times\mathbf{h}_e)\right]\right)$$

$$\mathbf{h}_e = h_{ac}\cos\Omega t\ \ \hat{\mathbf{e}}_x + (h_{ac}\ sin\Omega t + \Gamma_{ij}\epsilon_J\cos\theta)\ \ \hat{\mathbf{e}}_y + (1 + \Gamma_{ji}\epsilon_J\cos\theta)\ \ \hat{\mathbf{e}}_z$$

and

$$\Gamma_{ij} = \frac{\sin(\phi_{si}m_j)}{m_i(\phi_{si}m_j)}\left[\cos(\phi_{sj}m_i) - \frac{\sin(\phi_{sj}m_i)}{(\phi_{sj}m_i)}\right]\delta_J$$

where i=y, j=z and

$$\delta_J = \begin{cases} 0 : \mathbf{h}_e = \mathbf{h}_{ac} + \mathbf{h}_0 \\ 1 : \mathbf{h}_e = \mathbf{h}_{ac} + \mathbf{h}_0 + \mathbf{h}_s \end{cases}$$

llinearized equations →

❑ In case of high-frequency magnetic susceptibility the linearized LLG gives

$$m_y = \frac{1}{\Delta}\left[\frac{-\Upsilon_1\cos(\Omega t) + \Upsilon_2\sin(\Omega t)}{D}\right]$$

$$\Upsilon_1 = 2\alpha\frac{\Omega^2}{\Omega_0^2}$$

$$\Upsilon_2 = \left(1 - (1-\alpha^2)\frac{\Omega^2}{\Omega_0^2}\right)$$

$$D = \left(1 - (1+\alpha^2)\frac{\Omega^2}{\Omega_0^2}\right)^2 + 4\alpha^2\frac{\Omega^2}{\Omega_0^2}$$

$$\Delta = 1 + \frac{\phi_{sz}^2\epsilon_J(1 - (1 - \alpha^2\Omega^2/\Omega_0^2))}{3D}\cos\theta$$

RCSJ with β_c= 0 reads as

$$I/I_c^0 = \frac{\sin(\phi_{sy}m_z)\sin(\phi_{sz}m_y)}{(\phi_{sy}m_z)(\phi_{sz}m_y)}\sin\theta + \frac{d\theta}{dt}$$

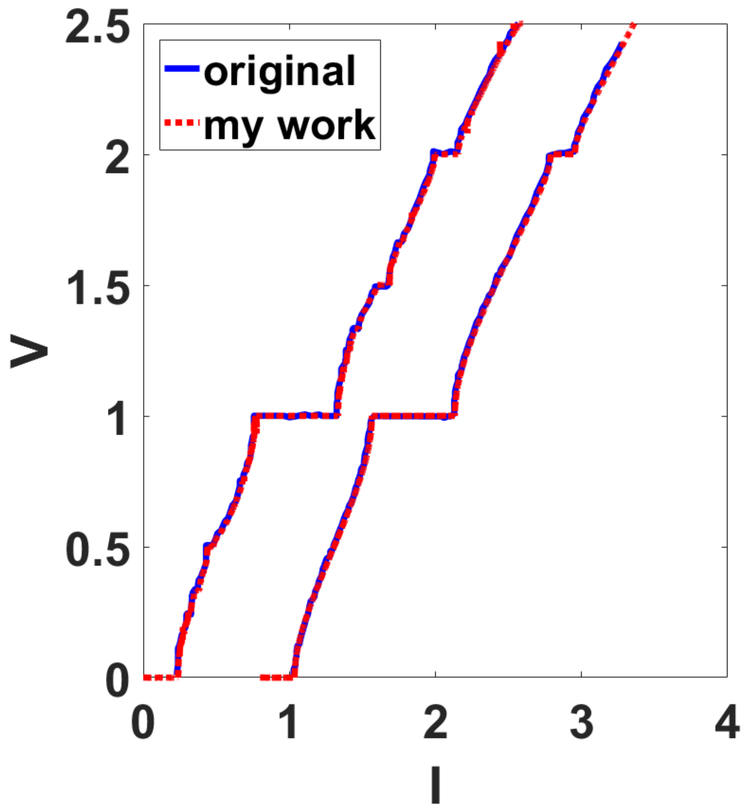
RCSJ with β_c= 0 reads as

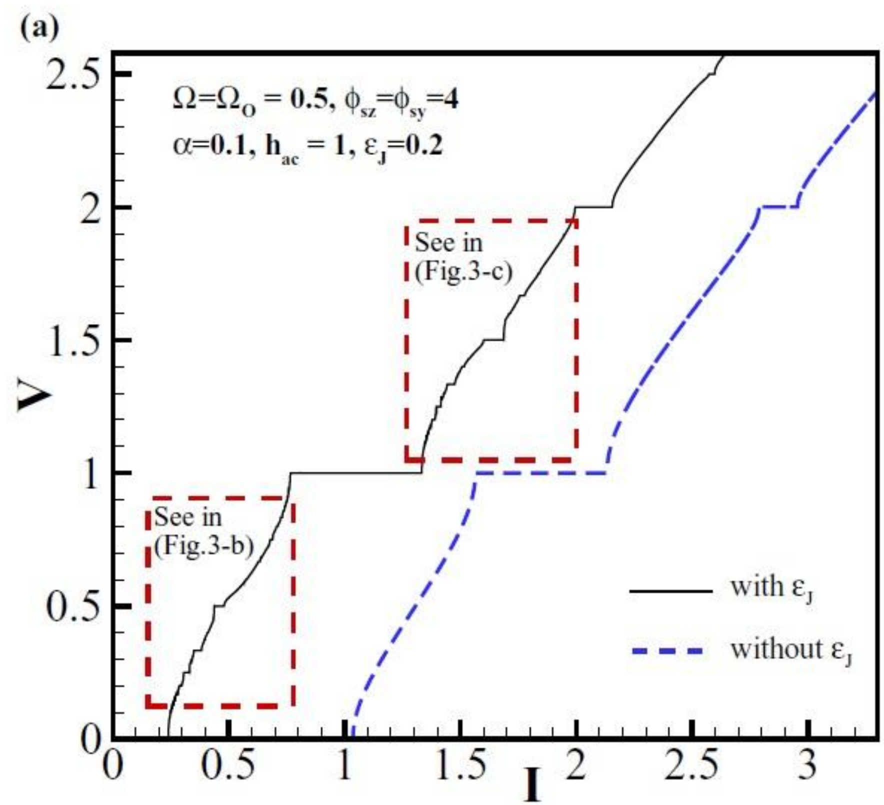
$$I/I_c^0 = \frac{\sin(\phi_s m_y)}{\phi_s m_y}\sin\theta + \frac{d\theta(\tau)}{d\tau}$$

Results

Devil's staircases in the IV characteristics of superconductor/ferromagnet/superconductor Josephson junctions

M. Nashaat,[1,2] A. E. Botha,[3] and Yu. M. Shukrinov[2,3,4,*]
[1]*Department of Physics, Cairo University, Cairo, 12613, Egypt*
[2]*BLTP, JINR, Dubna, Moscow Region, 141980, Russia*
[3]*Department of Physics, University of South Africa, Science Campus, Private Bag X6, Florida Park 1710, South Africa*
[4]*Dubna State University, 141982 Dubna, Russian Federation*





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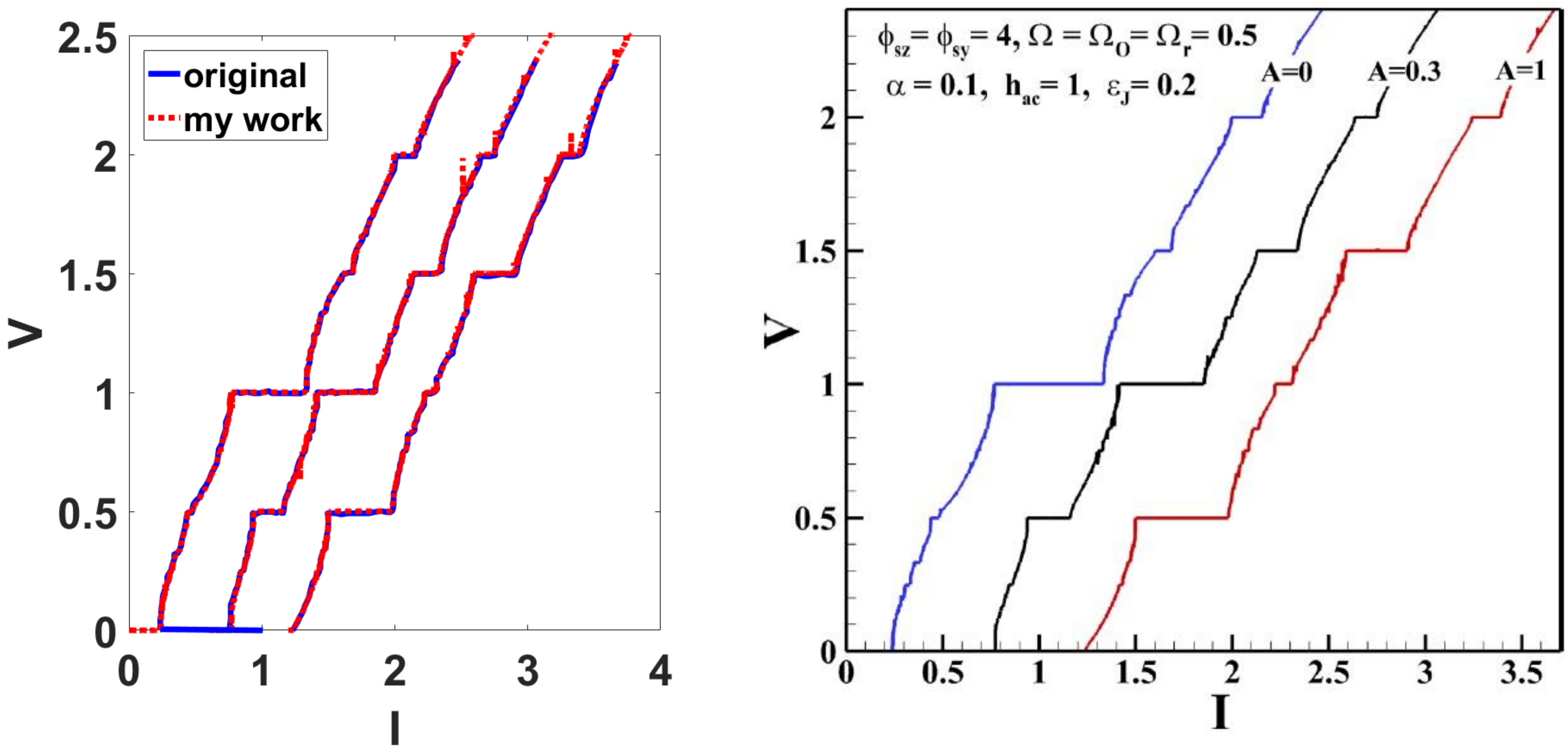

conclusion

- I trained on new topic (superconductor)
- I will trained on C++ and simulation methods
 through this school
- I got a will knowledge about the the equation of SFS
- I validated a previous published paper

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K. Kulikov


Thank you!