Bogoliubov Laboratory of Theoretical Physics

Numerical methods in theory of topological solitons

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Aims of the project:

- To study knot solutions (hopfions) in 3+1 dimension space-time in the scalar field theories;
- To obtain numerical solution in the Faddeev-Skyrme scalar model with exact topological charge

Lagrangian of the Faddeev-Skyrme model

$$\mathcal{L} = \left(\partial_\mu \phi^a\right)^2 - \frac{1}{2}\left(\varepsilon_{abc}\phi^a \partial_\mu \phi^b \partial_\nu \phi^c\right)^2 - U(\phi)$$

where ϕ^a – scalar triplet ($a = 1,2,3$) which constrained to the surface of a unit sphere $\phi^a \cdot \phi^a = 1$;

$U(\phi)$ - potential term Which does not contain the derivatives.

Let us consider the simplest case $U(\phi) = 0$. The normalized static energy functional of the model:

$$E = \frac{1}{32\pi^2\sqrt{2}} \int d^3x \left\{ (\partial_i \phi^a)^2 + \frac{1}{2}\left(\varepsilon_{abc}\phi^a \partial_i \phi^b \partial_j \phi^c\right)^2 \right\}$$

Hopf map

- The scalar field of the Faddeev-Skyrme model is a map from the compactified coordinate space S^3 to the target space S^2.

- Rational map parametrization:

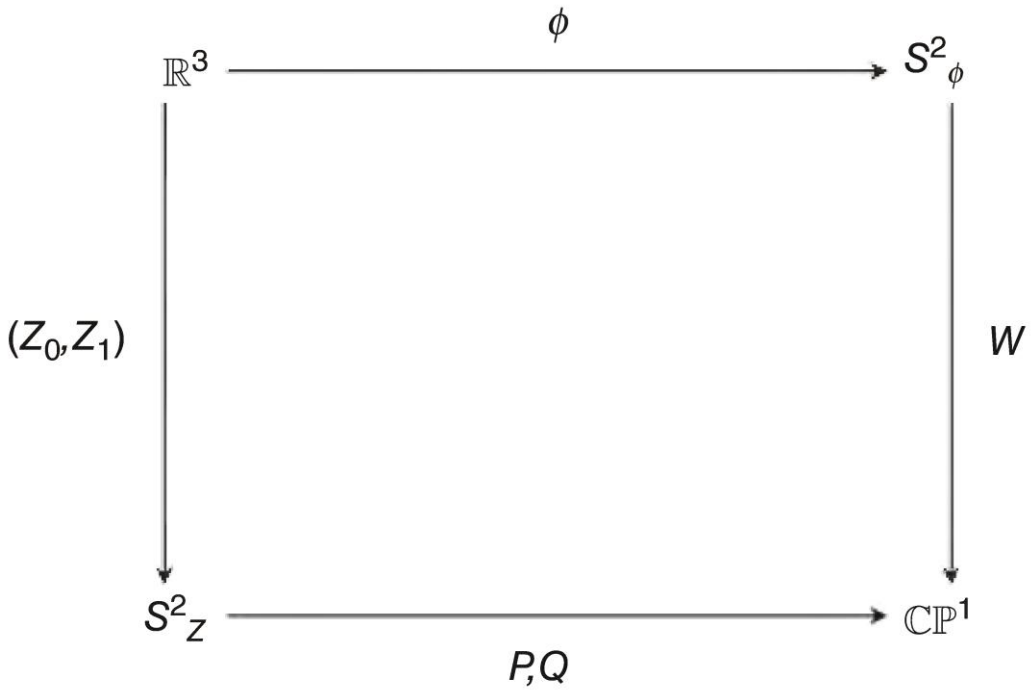
$$(Z_1, Z_0) = (\sin f(r) \sin\theta e^{i\varphi};\ \ \cos f(r) + i \sin f(r)\ \cos\theta)$$

where $f(r)$ - monotonically decreasing function

$(f(0) = \pi, f(\infty) = 0)$.

$$W(Z_1, Z_0) = \frac{\phi_1 + i\phi_2}{1 + \phi_3} = \frac{P(Z_1, Z_0)}{Q(Z_1, Z_0)}$$

Here the polynomials $P(Z_1, Z_0)$ and $Q(Z_1, Z_0)$ have no common factors and no common roots on the two-sphere S^2.



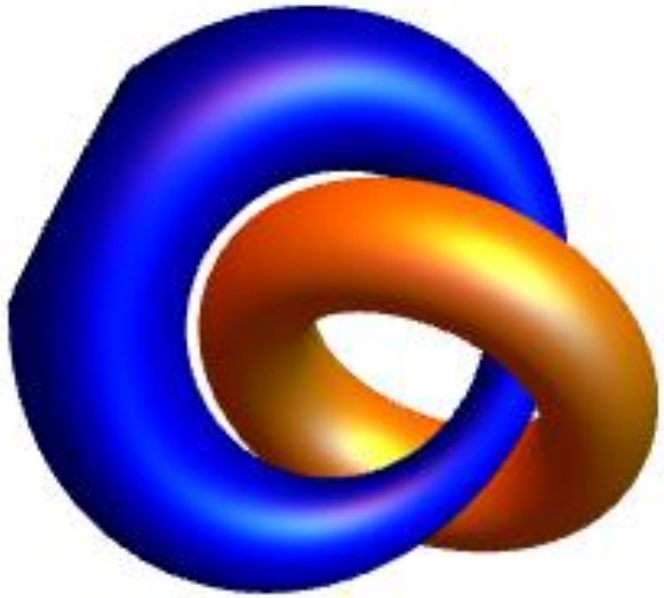
Method of random rotations

Aim of the method: To minimize energy of hopfion.

1. Choose random point (i,j,k);
2. Rotate all ϕ^a in a cube (i,i+n) (j,j+n) (k,k+n) on the infinitely small angle along the random axis with Gauss distribution;
3. If energy decreased then accept changes, else new iteration in new random point;

Results

- Hopf index Q=1

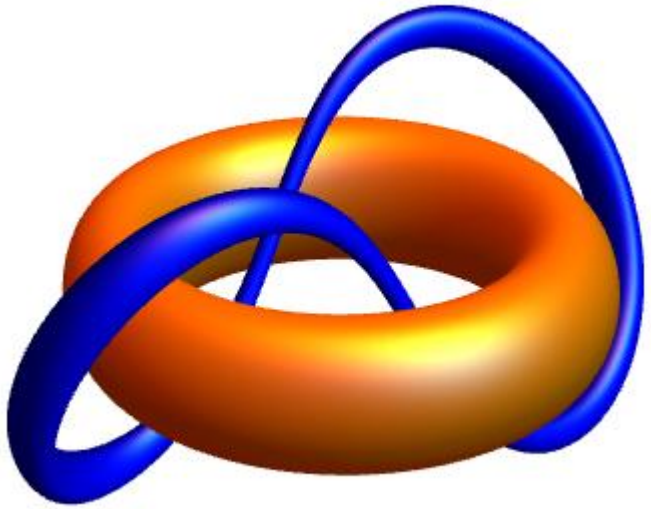


Initial isosurfaces for ϕ_1 and ϕ_3

Initial energy: $E_{in} = 1.854$
Global minima of energy: $E_{min} = 1.230$
Estimated energy: $E_{est} = 1.231$
Estimated charge: $Q = 0.895$

Results

- Hopf index Q=2



Initial isosurfaces for ϕ_1 and ϕ_3

Initial energy: $E_{in} = 2.593$
Global minima of energy: $E_{min} = 1.968$
Estimated energy: $E_{est} = 1.971$
Estimated charge: $Q = 1.912$

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