Numerical simulation of homogeneous and isotropic universe given by Friedmann equations

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Outline

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- Newtonian cosmology
- Friedmann equations as a function of time
- The fluid equation
- The equation of state
- Redshift
- Friedmann equations as a function of redshift
- Numerical solutions
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Aim of the work

This project aims to provide a mathematical framework for modeling the Universe. By analyzing cosmological data we compute a set of parameters that together with the mathematical model, describe the past and future evolution of the Universe.

Hubble law

- Hubble's law is a statement of a direct correlation between the distance to a galaxy and its recessional velocity as determined by the redshift.

$$v = H * R \qquad (1)$$

Where:

v recessional velocity in km/sec

R distance to the Galaxy in Mpc

H expansion rate (Hubble parameter)

i. its value tells us how fast the Universe is expanding.

ii. Equal fractional rate of change of the scale factor i.e., $H = \frac{\dot{a}}{a}$

- Hubble's Law shows that the Universe is expanding in a systematic way i.e., The more distant a galaxy is, the faster it appears to be moving away from us.

Newtonian cosmology

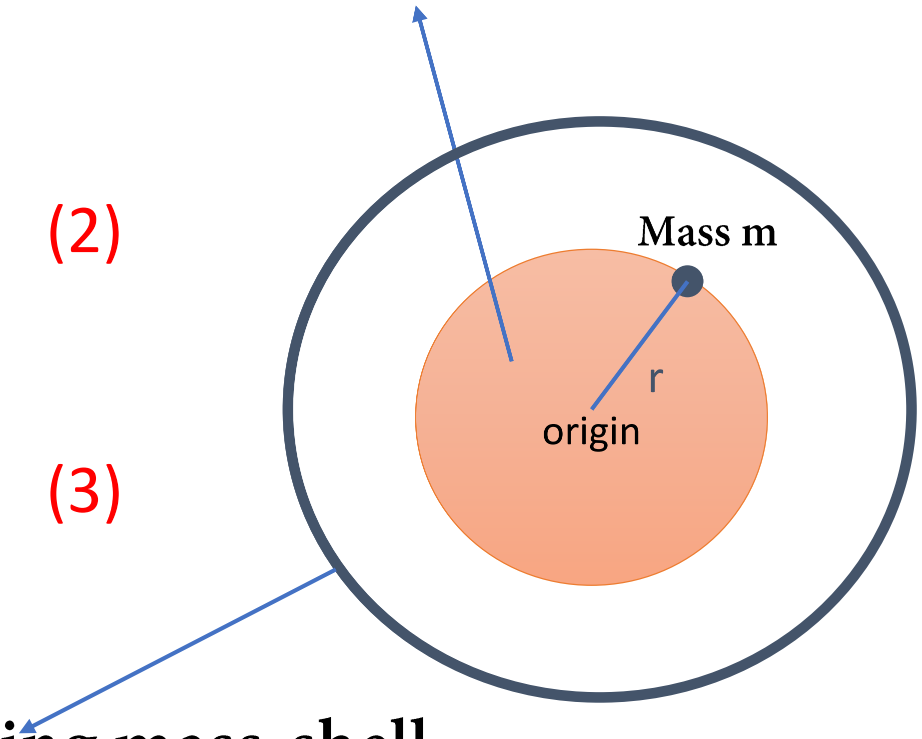
One can describe the expansion of the universe, without being completely rigorous, using simple Newtonian gravity, instead of using the equations of General Relativity

- Newton's theorem

$$F = G\frac{mM}{R^2} = -m\ddot{R} \qquad (2)$$

$$V = -G\frac{mM}{R} \qquad (3)$$

Contributing mass M



Non-contributing mass-shell

Friedmann equations

The Friedmann equations start with the simplifying assumption that the universe is spatially homogeneous and isotropic.

Aleksandr Friedmann
(1888-1925)

$$H^2 + \frac{k}{a^2} = \frac{8\pi}{3} G\varepsilon \qquad (4)$$

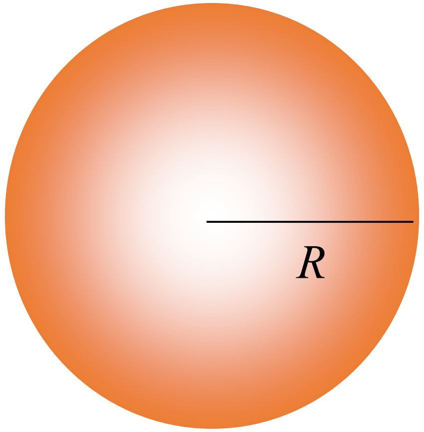
$$\ddot{a} = -\frac{4\pi}{3} G(\varepsilon + 3p)a \qquad (5)$$

Describes the expansion of the universe and can therefore be considered the most important equations in cosmology. Also Friedmann equations have three unknown parameters: ε , p and a

Fluid equation

The density of the sphere is given by

$$\varepsilon(t) = \frac{M}{4\pi/_{3}\,R(t)^3} \qquad (6)$$



Then the energy density in Volume V is

$$E = \varepsilon V;$$

$$dE = V d\varepsilon + \varepsilon dV$$

According to 2nd law of thermodynamics:

$$dQ = dE + PdV;$$

$$V \propto a^3 \text{ So } {}^{dV}/_{V} = 3\,{}^{da}/_{a}\,;$$

$$\dot{\varepsilon} = -3H(\varepsilon + p) \qquad (7)$$

This is called the *Fluid equation*. It tells us how the energy density of universe changes with time.

Equation of state

In cosmology, the equation of state of a perfect fluid is characterized by a dimensionless number, equal to the ratio of its pressure to its energy density :

$$p = w\varepsilon \qquad (8)$$

If a is the scale factor then

$$\varepsilon \propto a^{-3(1+w)}$$

The *equation of state* depends on the substance under consideration. For example, for ordinary matter

$$w = 0; \;\; \varepsilon \propto a^{-3}$$

and for radiation

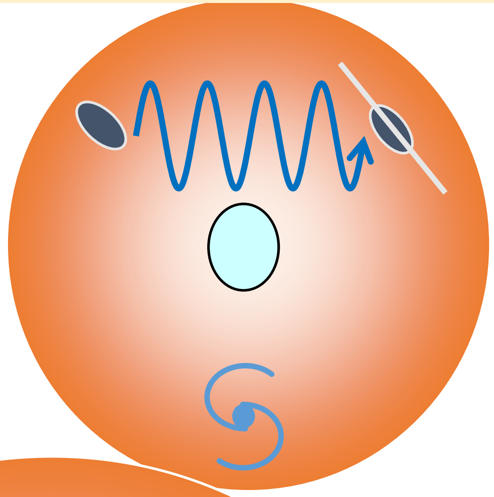
$$w = 1/3; \;\; \varepsilon \propto a^{-4}$$

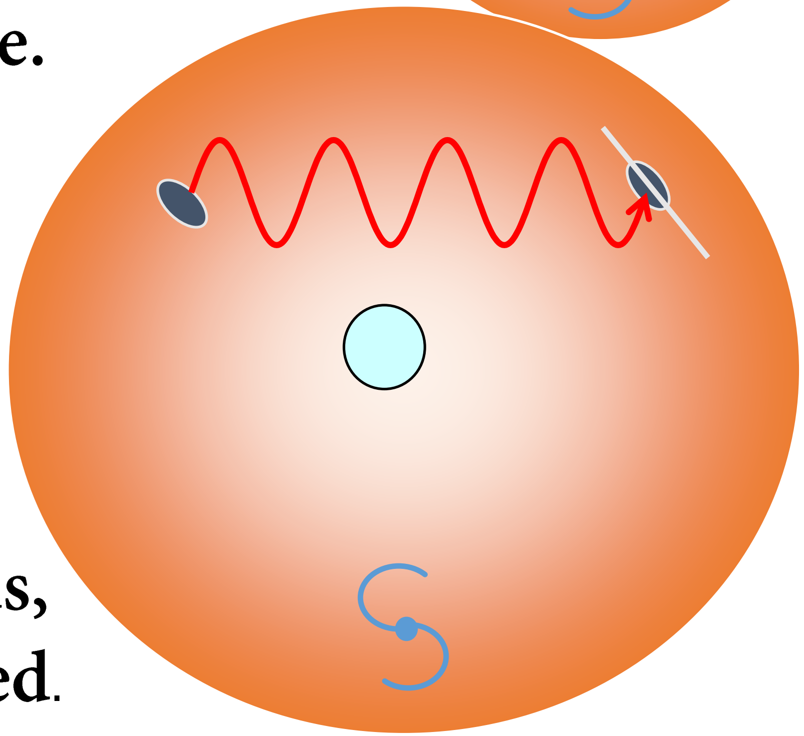
Cosmological Redshift

- Expansion of space :
  - Stretches light into longer wavelengths
  - The greater the distance, the greater the stretching
- Result:
  - The redshift of an object gets larger with distance.
  - Just what Hubble actually measured
- The Redshift equation:

$$1 + z = \frac{a_o}{a(t)} \qquad (9)$$

- The scale factor is increasing as time passes, thus, z is positive and distant galaxies appear redshifted.





Friedmann equations

To be able to utilize measured data we use the concept of redshift, and rewrite Friedmann equations as function of redshift instead of time:

$$\frac{dH}{dz} = \frac{1}{z+1}\left\{H + \frac{4\pi}{3H}G(\varepsilon + 3p)\right\} \quad (10)$$

$$\frac{da}{dz} = -\frac{a}{z+1} \quad (11)$$

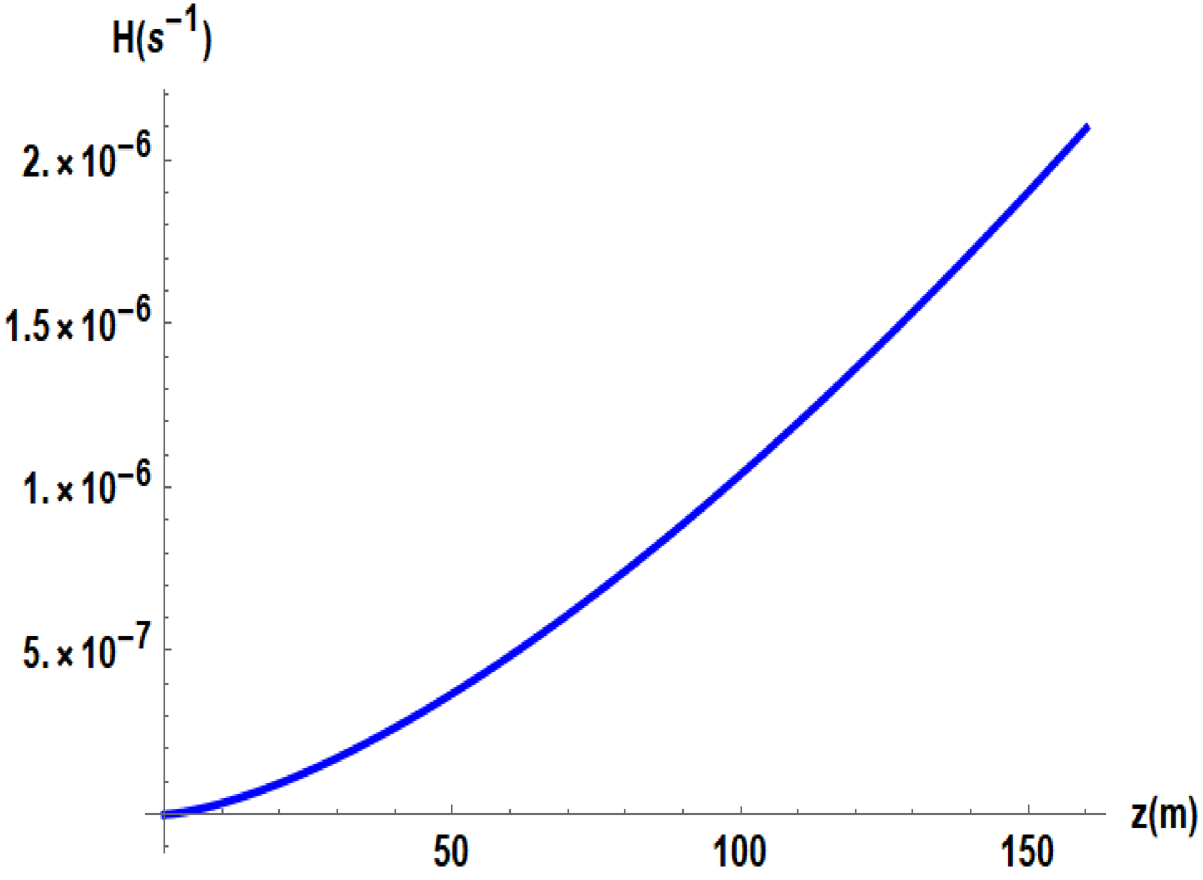
$$\frac{d\varepsilon}{dz} = \frac{3(\varepsilon + p)}{z+1} \quad (12)$$

Then we will numerically solve this system of differential equations

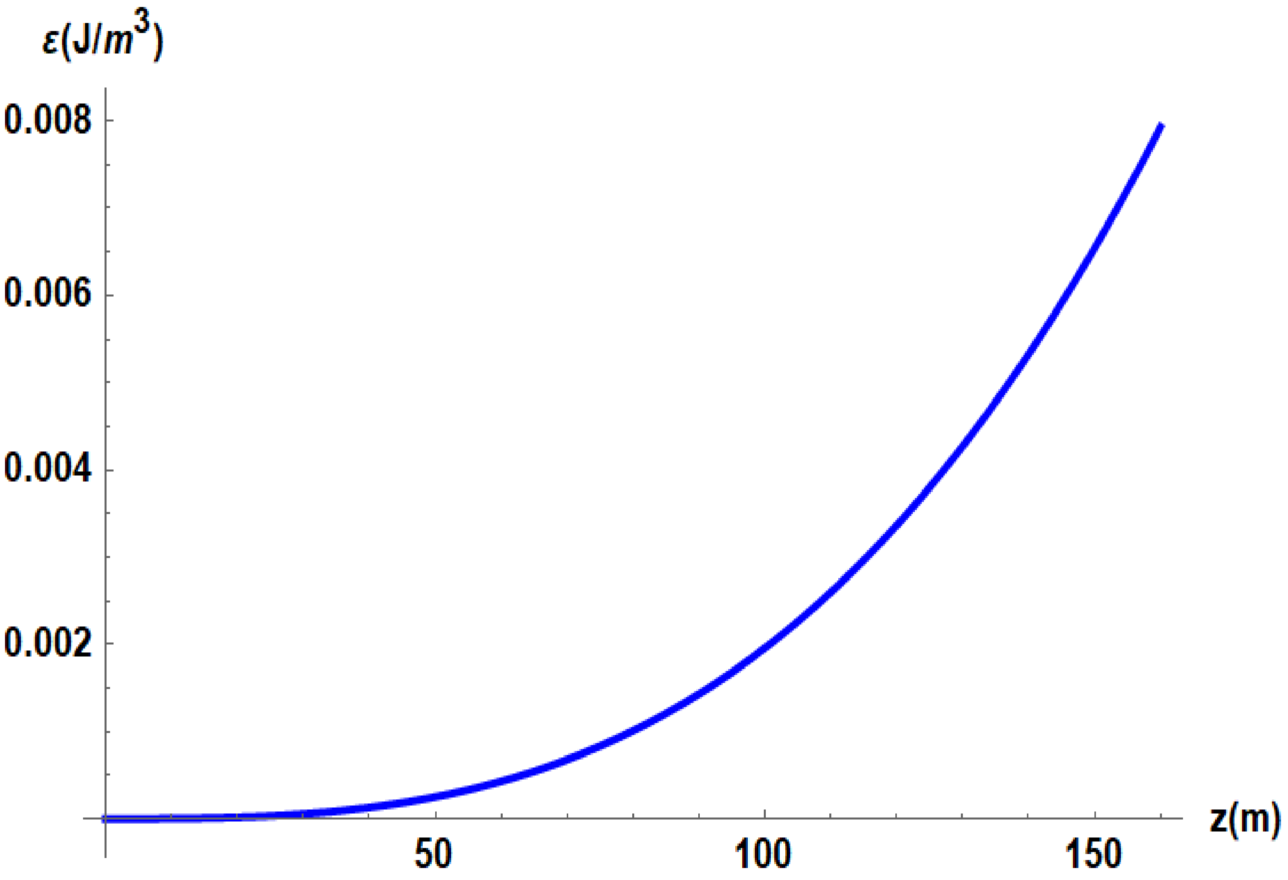
Numerical solutions

Dust filled universe

W = 0



(a) Hubble parameter as a function of redshift

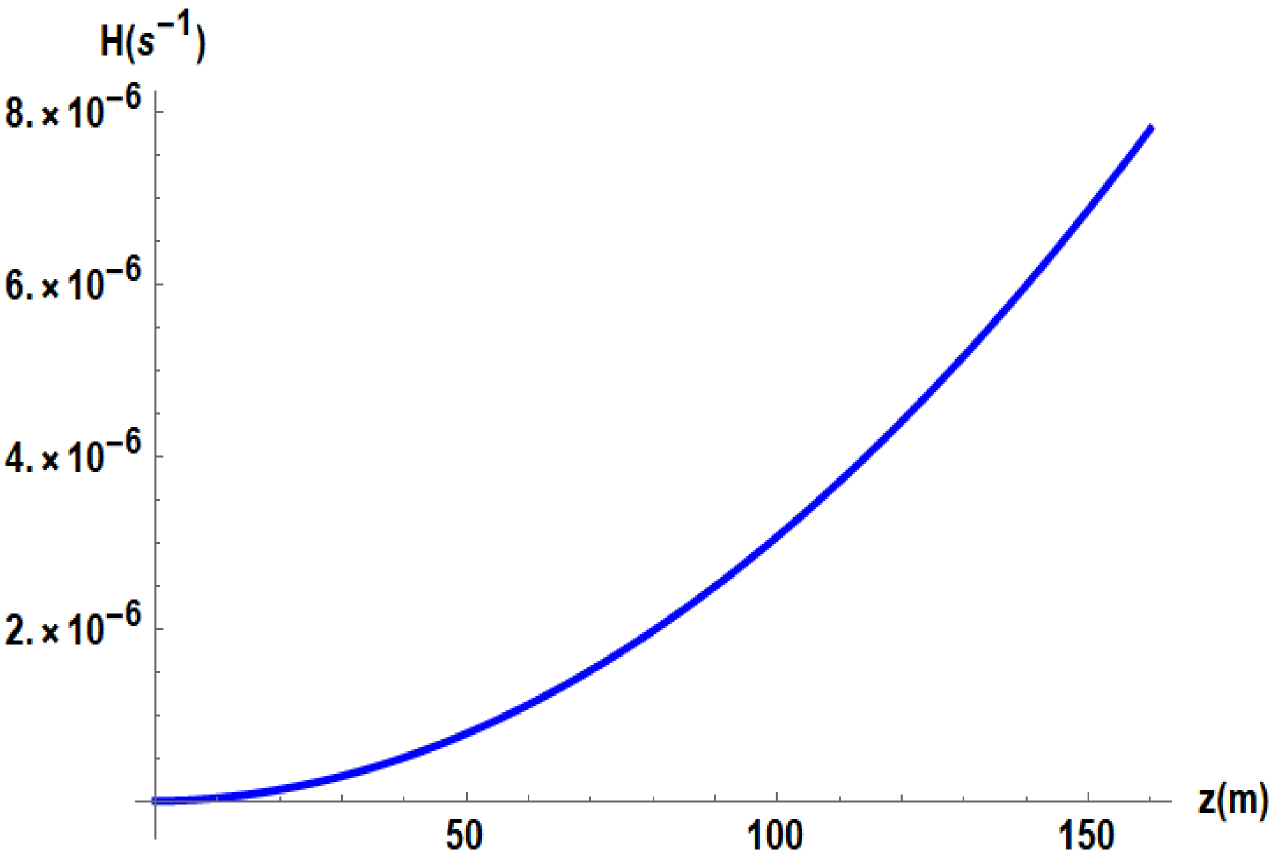


(b)Energy density as a function of redshift

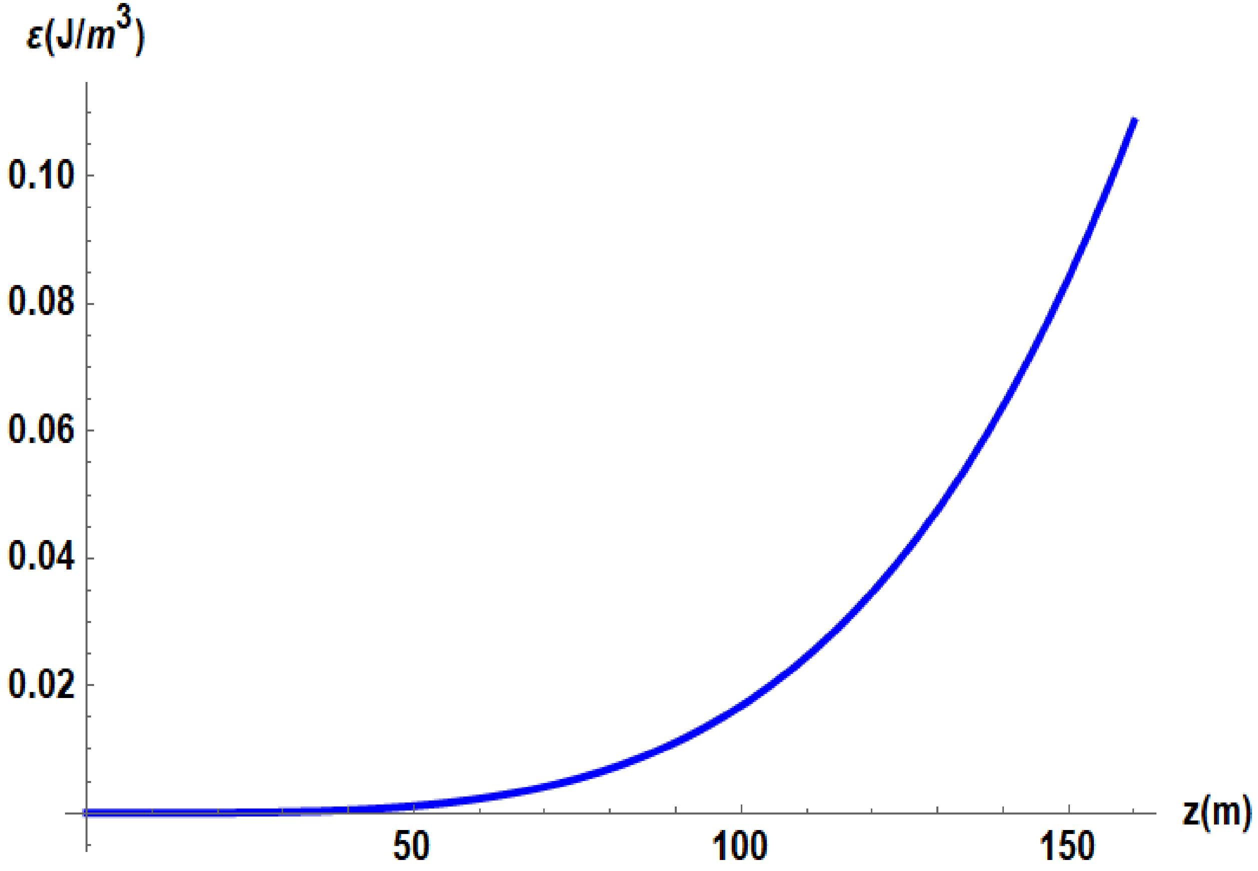
Numerical solutions

radiation filled universe

W = 1/3



(a) Hubble parameter as a function of redshift



(b) Energy density as a function of redshift

Resources

- SAI Supernova Catalogue

http://stella.sai.msu.su/sncat/download.html

- Physical Foundations of Cosmology by Viatcheslav Mukhanov

Thank you
for your attention