

# Study of heavy-ion elastic scattering within classical and quantum optical model



By:

**Ragab Ahmed Abdelghany**

*Physics Department, Faculty Of Science,  
Al-Azhar University, Egypt.*

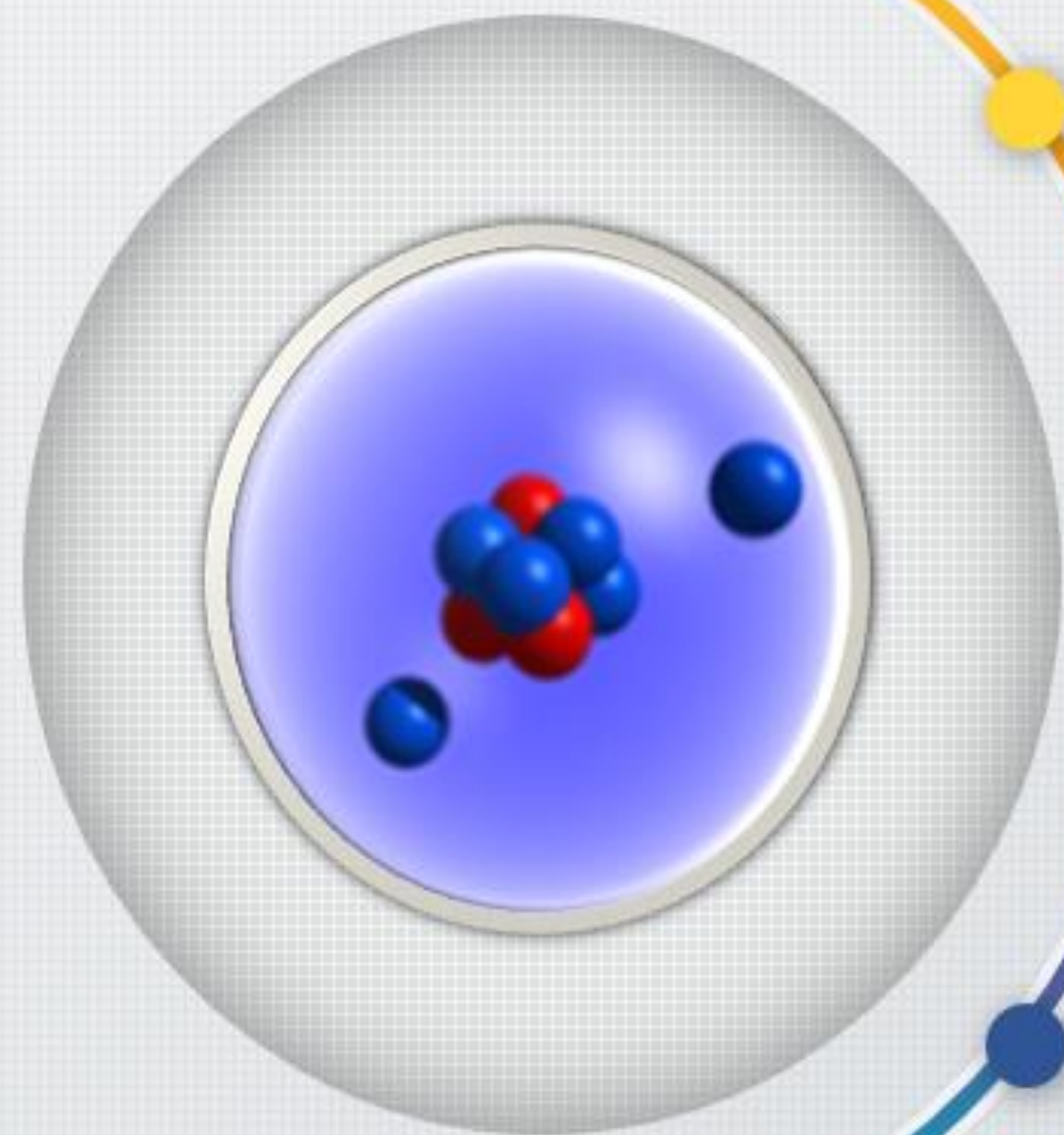
Supervised by:

**Vladimir Rachkov**

*Flerov Laboratory of Nuclear Reactions.*



## Contents:



1 Aim of the work

2 Introduction

3 Theoretical Part

4 Practical part

5 p and n scattering

6 Energy dependence

7 Conclusions



# Aim of the work

---

1- The project aims to study of the behavior of the elastic scattering differential cross section for different energies and different nucleon-target combinations

2- Derivation of expressions for:  
**a-** partial wave expansion of a plane wave.  
**b-** relation between the elastic cross section and phase shifts.  
**c-** relation between the scattering amplitude and the phase shifts

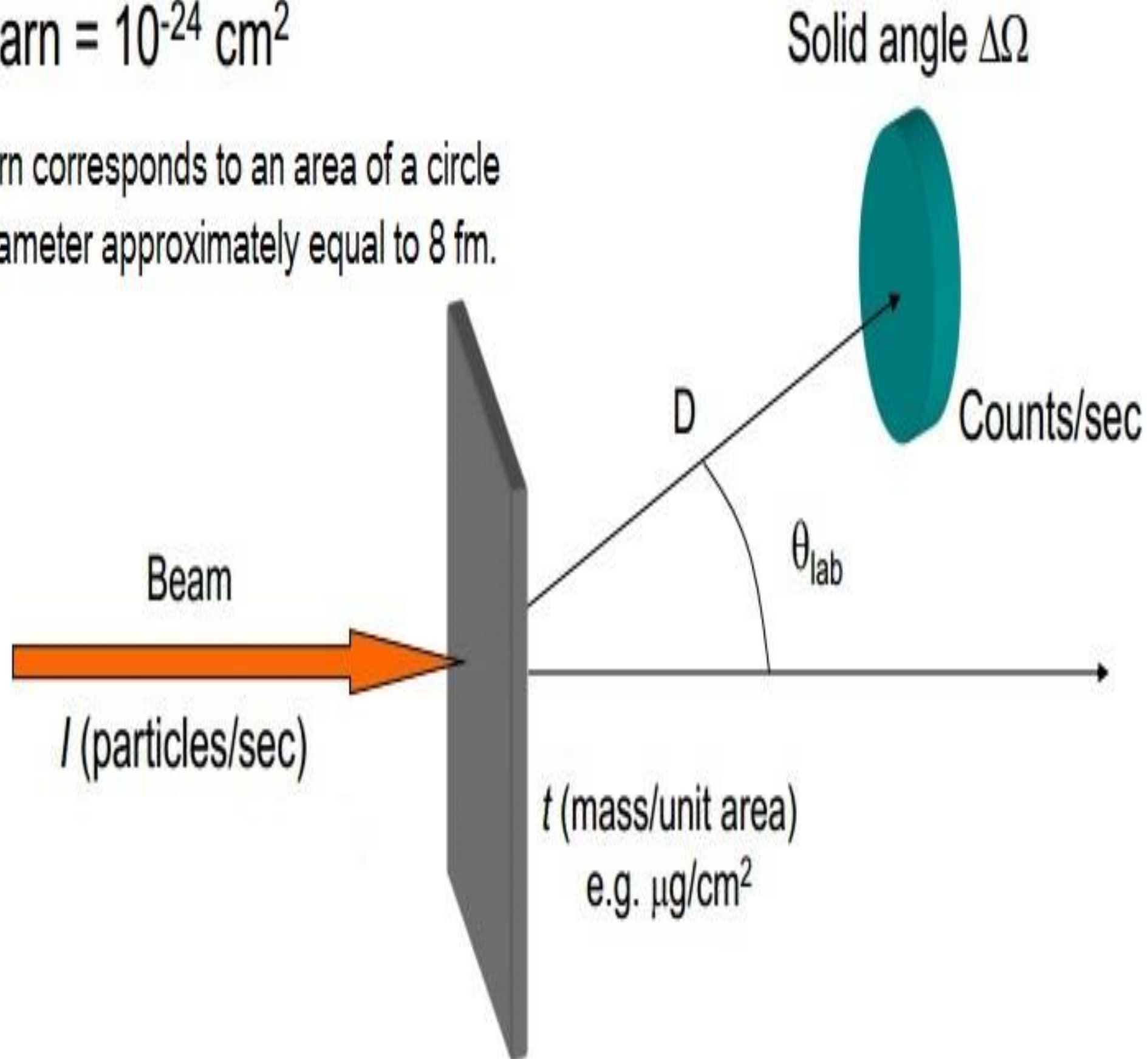
3- Digitizing experimental data from the graphs of articles using GSYS program and Importing data in the NRV project for low energy codes

4- Search for the optical potential parameters which produce best fit with the experimental data. And finally Interpreting the results

# Intro:

$$1 \text{ barn} = 10^{-24} \text{ cm}^2$$

1 barn corresponds to an area of a circle of diameter approximately equal to 8 fm.



## The nuclear reactions:

A large fraction of our knowledge about the internal structure of nuclei is came from the nuclear reactions.

$$N_{\alpha}(\Omega, \Delta\Omega) \propto (\Delta\Omega \cdot n \cdot j)$$

$$N_{\alpha}(\Omega, \Delta\Omega) = (\Delta\Omega \cdot n \cdot j) \cdot (\text{proportionality constant})$$

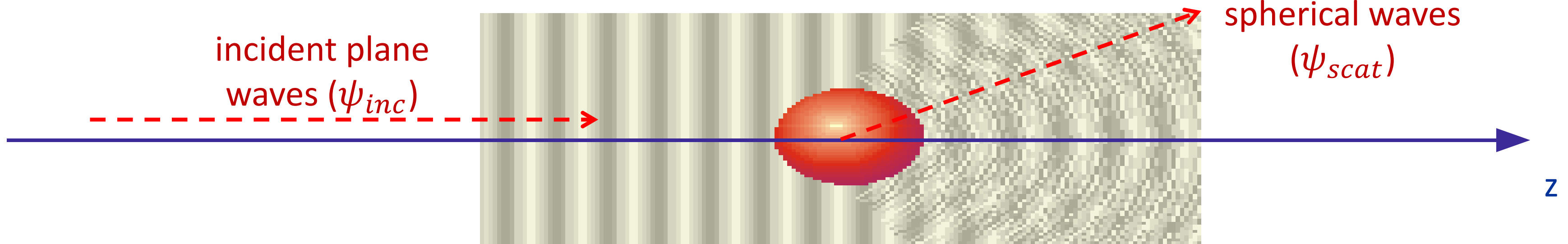
$$N_{\alpha}(\Omega, \Delta\Omega) = (\Delta\Omega \cdot n \cdot J) d\sigma_{\alpha}(\Omega)/d\Omega$$

$$\frac{d\sigma_{\alpha}(\Omega)}{d\Omega} = \frac{N_{\alpha}(\Omega, \Delta\Omega)}{\Delta\Omega \cdot n \cdot J}, \rightarrow \text{Differential Cross Section}$$

$$\sigma_{\alpha} = \int d\Omega \left[ \frac{d\sigma_{\alpha}(\Omega)}{d\Omega} \right] \rightarrow \text{The Reaction Cross Section}$$

In the center of mass frame, the problem of scattering between two particles (p and t) is reduced to the scattering of a particle of mass ( $\mu$ ) by a finite range central potential  $V(r)$ .

$$\left[ -\frac{\hbar^2}{2\mu} (\nabla^2 + k^2) + V(\vec{r}) \right] \psi(\vec{r}) = 0 \quad (1)$$



The solution of this equation is:

$$\psi(\vec{r}) = \psi_{inc}(\vec{r}) + \psi_{sca}(\vec{r}),$$

where

$$\psi_{inc}(\vec{r}) = A e^{ik_0 z}.$$

$$\psi_{sca}(\vec{r}) = A f(\theta, \phi) \frac{e^{ikr}}{r} \quad , r \gg a$$

➔ The most general solution of the Schrödinger equation (1) is

$$\psi(\vec{r}) = \frac{u_{El}(r)}{r} Y_{lm}(\theta, \phi)$$

➔ When the potential is central, the scattered wave function don't depend on ( $\phi$ ) and hence  $m=0$ . Thus:

$$\psi(\vec{r}) = \frac{u_{El}(r)}{r} Y_{l0}(\theta)$$

➔ Where  $u_{El}(r)$  is the solution of the radial wave function, which is expressed as the following in case of free particles ( $V(r) = 0$ ):

$$\left( -\frac{d^2}{dr^2} + \frac{l(l+1)}{r^2} \right) u_{El}(r) = k^2 u_{El}(r)$$

➔ The general solution of this equation is given by a linear combination of the spherical Bessel and Neumann functions

$$u_{El} = A_l \rho j_l(\rho) + B_l \rho n_l(\rho), \quad \rho = kr$$

➔ Thus one can generate plane waves from a linear combination of the spherical Bessel and Neumann functions:

$$e^{i\vec{k}\cdot\vec{r}} = e^{ikr\cos\theta} = \sum_l a_l (A_l j_l(\rho) + B_l n_l(\rho)) Y_{l0}(\theta)$$

➔ The most general solution of the Schrödinger equation (1) is

$$\psi(\vec{r}) = \frac{u_{El}(r)}{r} Y_{lm}(\theta, \phi)$$

➔ When the potential is central, the scattered wave function don't depend on  $(\phi)$  and hence  $m=0$ . Thus:

$$\psi(\vec{r}) = \frac{u_{El}(r)}{r} Y_{l0}(\theta)$$

➔ Where  $u_{El}(r)$  is the solution of the radial wave function, which is expressed as the following in case of free particles ( $V(r) = 0$ ):

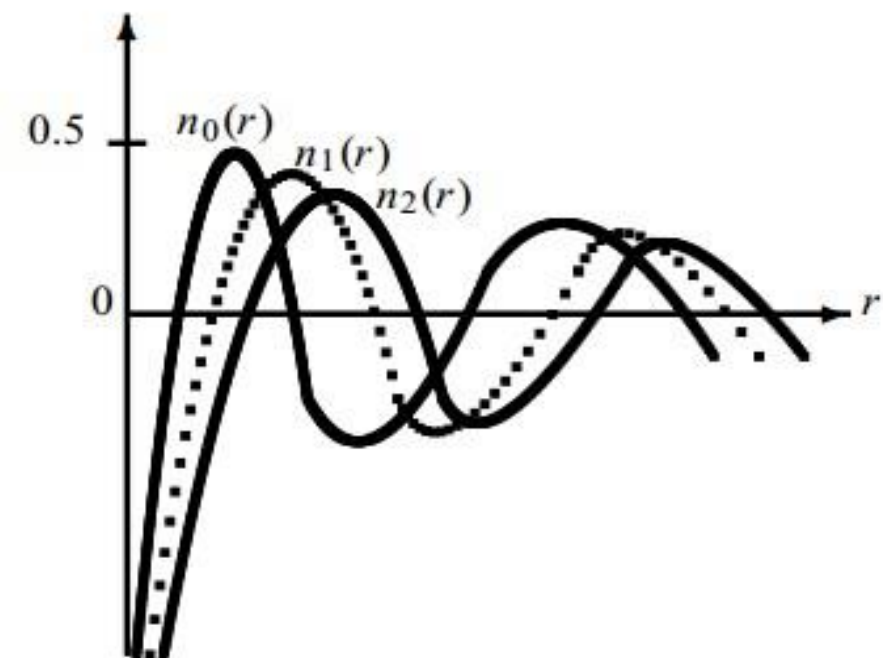
$$\left( -\frac{d^2}{dr^2} + \frac{l(l+1)}{r^2} \right) u_{El}(r) = k^2 u_{El}(r)$$

➔ The general solution of this equation is given by a linear combination of the spherical Bessel and Neumann functions

$$u_{El} = A_l \rho j_l(\rho) + B_l \rho n_l(\rho), \quad \rho = kr$$

➔ Thus one can generate plane waves from a linear combination of the spherical Bessel and Neumann functions:

$$e^{i\vec{k}\cdot\vec{r}} = e^{ikrcos\theta} = \sum_l a_l (A_l j_l(\rho) + B_l \cancel{n_l(\rho)}) Y_{l0}(\theta)$$



➔ A substitution of  $a_l$  and  $j_l(\rho)$  into the previous equation:

$$e^{ikr} \cong \frac{\sqrt{4\pi}}{k} \sum_l \sqrt{2l+1} i^l Y_{l0} \frac{1}{2i} \left( \frac{e^{i(kr - \frac{l\pi}{2})}}{r} - \frac{e^{-i(kr - \frac{l\pi}{2})}}{r} \right), r \gg a \quad (1)$$

➔ Thus the scattered wave function is going to be

$$\begin{aligned} \psi(\vec{r}) &= \psi_{inc}(\vec{r}) + \psi_{sca}(\vec{r}) \\ &= \frac{\sqrt{4\pi}}{k} \sum_l \sqrt{2l+1} i^l Y_{l0} \frac{1}{2i} \left( \frac{e^{i(kr - \frac{l\pi}{2})}}{r} - \frac{e^{-i(kr - \frac{l\pi}{2})}}{r} \right) + f_k(\theta) \frac{e^{ikr}}{r} \quad r \gg a \end{aligned} \quad (2)$$

➔ The right hand side of equation (1) has to take the following form:

$$\psi(\vec{r}) = \frac{\sqrt{4\pi}}{k} \sum_l \sqrt{2l+1} i^l Y_{l0} \frac{1}{2i} \left( \frac{e^{-i(kr - \frac{l\pi}{2}) + 2i\delta_l}}{r} - \frac{e^{-i(kr - \frac{l\pi}{2})}}{r} \right) \quad (3)$$

➔ Where  $\delta_l$  is the phase shifts. By comparison (1) and (2) one can find:

$$f_k(\theta) = \frac{\sqrt{4\pi}}{k} \sum_l \sqrt{2l+1} Y_{l0}(\theta) e^{i\delta_l} \sin(\delta_l). \quad \text{Finally} \rightarrow \frac{d\sigma}{d\Omega} = |f_k(\theta)|^2$$



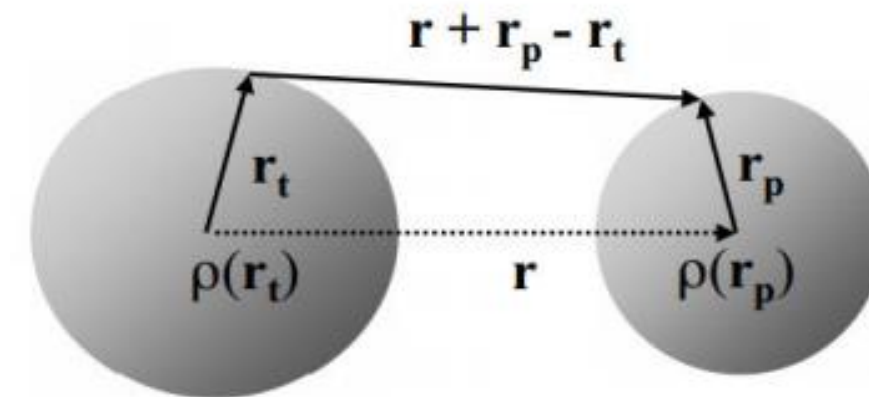
# The Optical Potential:

There are basically two ways to describe the optical potential:

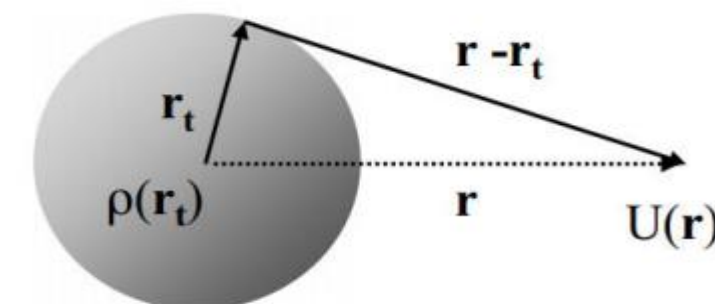
## (1) Phenomenological Optical potential

$$\begin{aligned}
 V(r) = & -V f_1(r) \quad \text{a real volume term,} \\
 & +iW_v f_2(r) \quad \text{an imaginary volume term,} \\
 & -iW_d \frac{d}{dr} f_3(r) \quad \text{an imaginary surface term,} \\
 & -V_{s.o} \frac{b}{r} \frac{d}{dr} f_3(r) \vec{\ell} \cdot \vec{\sigma} \quad \text{a real spin - orbit term,} \\
 & -iW_{s.o} \frac{b}{r} \frac{d}{dr} f_4(r) \vec{\ell} \cdot \vec{\sigma} \quad \text{an imaginary spin - orbit term} \\
 & +V_C(r) \quad \text{and Coulomb term.}
 \end{aligned}$$

## (2) Microscopic Folding potential approach



$$V_{DF}(r) = \int \int V_{NN}(\mathbf{r} + \mathbf{r}_P - \mathbf{r}_T) \rho_T(\mathbf{r}_T) \rho_P(\mathbf{r}_P) d\mathbf{r}_T d\mathbf{r}_P,$$



$$V_{SF}(r) = \int V_{NN}(\mathbf{r} - \mathbf{r}_T) \rho_T(\mathbf{r}_T) d\mathbf{r}_T.$$

# Practical part



**NRV knowledge base**

<http://nrv.jinr.ru>

The NRV web knowledge base is a unique interactive research system:

- Allows to run complicated computational codes
- Works NRV browser, which can be downloaded free
- Has graphical interface for preparation of input parameters and analysis of output results
- Combines computational codes with experimental databases on properties of nuclei and nuclear reactions
- Contains detailed description of models

The screenshot shows the main interface of the NRV knowledge base. At the top, it features the NRV logo and the title "Nuclear Reactions Video Low Energy Nuclear Knowledge Base". Below this is a navigation menu with four main categories: Nuclear Properties, Nuclear Models, Nuclear Decays, and Nuclear Reactions. Each category has a list of sub-topics and links to various models and codes. On the left side, there are three prominent green buttons: "Getting Started", "NRV Browser v 2.0.3 (19 Dec 2019)", and another "NRV Browser v 2.0.3 (19 Dec 2019)". A warning message about Java installation is also visible. The bottom of the page contains a disclaimer about the free use of resources and a request for official support.

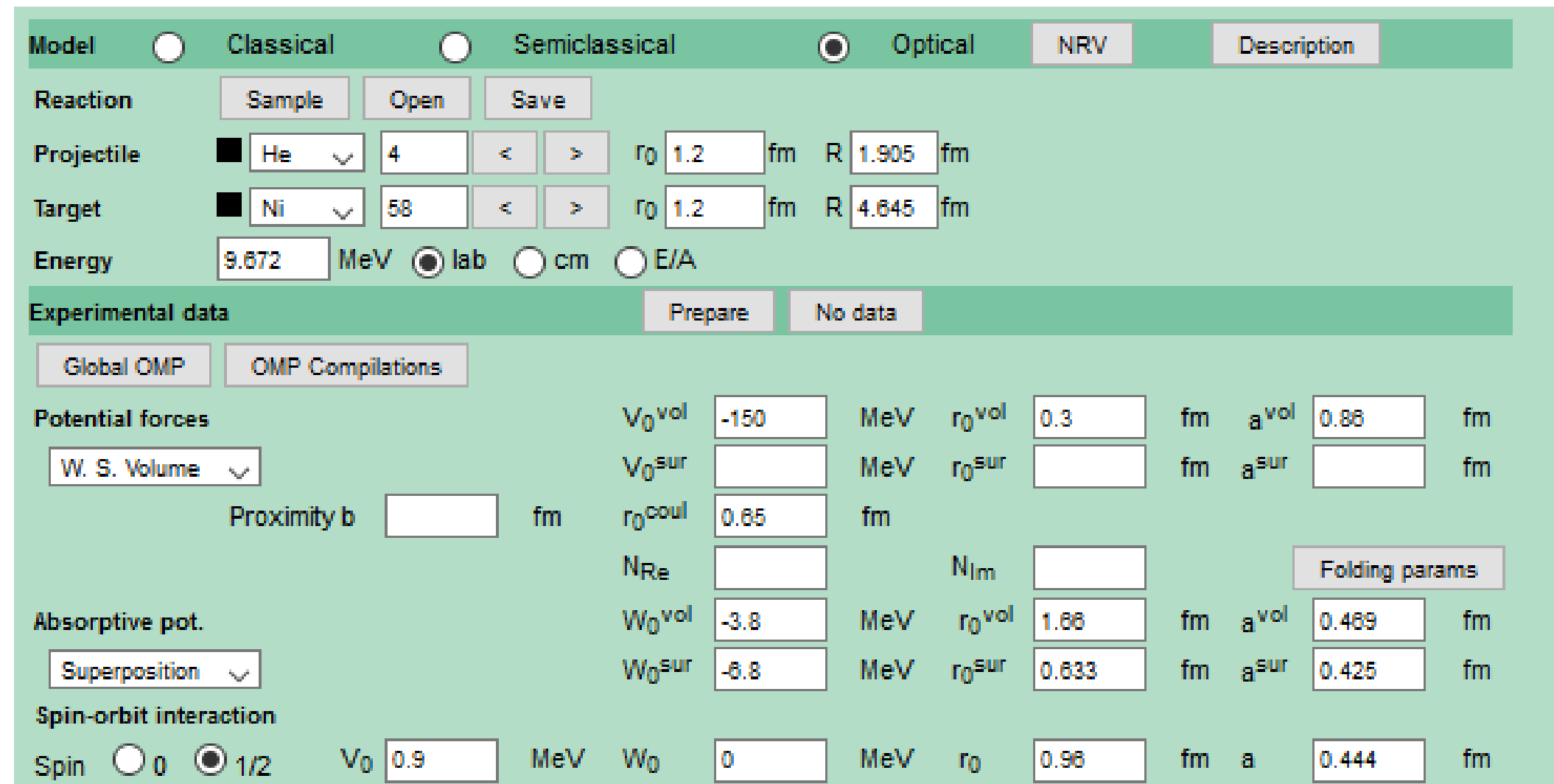
Nuclear Properties	Nuclear Models	Nuclear Decays	Nuclear Reactions
Nuclear Map Systematics JS	Shell Model	Alpha - decay	Available beams Stable beams: EU Institutions, FLNR (Dubna) U400, FLNR (Dubna) U400M RIBs: GANIL, MSU
Getting Started	Liquid Drop Model	Beta - decay	Elastic scattering Classical Semiclassical Optical Model (Tutorial in Russian) Phase analysis Experimental Data Java JS $d\sigma/d\Omega$
NRV Browser v 2.0.3 (19 Dec 2019)	Two-Center Shell Model	Fission	Inelastic Scattering: DWBA model (DWUCK4 code) Adiabatic rotational model (FRESCO code) Coulomb excitation Direct process (DWBA) Channel coupling Deep inelastic collision
NRV Browser v 2.0.3 (19 Dec 2019)		Decay of excited nuclei	Transfer reactions: Direct process (DWBA) Java JS Semiclassical approach (CRAZING code) 3-body classical model Two-nucleon transfer Massive transfer
Please, quote us as... Warning: sections marked by this icon or without any icons require installation of Java and a Java-compatible browser Java			Fragmentation EPAX v.3 Break-up (DWBA) Semiclassical model LISE++
All resources of the NRV Knowledge Base are free to use. We, nevertheless, need a support of our project by official establishment			Fusion Empirical model Channel Coupling Langevin equations Experimental Data Java JS

## Optical Model calculation with NRV OM code

### Main steps of calculation:

#### Physical

- Set projectile and target parameters (mass, spin, etc)
- Set the incident energy
- Set the parameters of the OM potential



The screenshot shows the NRV OM code interface with the following settings:

- Model:** Optical (selected), NRV
- Reaction:** Sample, Open, Save
- Projectile:** He, mass 4,  $r_0 = 1.2$  fm,  $R = 1.905$  fm
- Target:** Ni, mass 58,  $r_0 = 1.2$  fm,  $R = 4.845$  fm
- Energy:** 9.672 MeV, lab (selected), cm, E/A
- Experimental data:** Prepare, No data
- Potential forces:**
  - $V_0^{vol} = -150$  MeV,  $r_0^{vol} = 0.3$  fm,  $a^{vol} = 0.88$  fm
  - $V_0^{sur} =$  (empty),  $r_0^{sur} =$  (empty) fm,  $a^{sur} =$  (empty) fm
  - Proximity b = (empty) fm,  $r_0^{coul} = 0.85$  fm
  - $N_{Re} =$  (empty),  $N_{Im} =$  (empty)
- Absorptive pot.:**
  - $W_0^{vol} = -3.8$  MeV,  $r_0^{vol} = 1.66$  fm,  $a^{vol} = 0.469$  fm
  - $W_0^{sur} = -8.8$  MeV,  $r_0^{sur} = 0.633$  fm,  $a^{sur} = 0.425$  fm
- Spin-orbit interaction:**
  - Spin: 1/2 (selected), 0
  - $V_0 = 0.9$  MeV,  $W_0 = 0$  MeV,  $r_0 = 0.98$  fm,  $a = 0.444$  fm

## Optical Model calculation with NRV OM code

### Main steps of calculation:

#### Numerical

- Set the radial step for integration
- Set the maximum radius  $R$  for integration
- Set the maximum angular momentum  $L$

Integration parameters		Default values of int.parameters		for classical model	
Initial angle	<input type="text" value="1"/> deg.	Partial waves:		$b_1$	<input type="text" value="0"/> fm
Maximal angle	<input type="text" value="179"/> deg.	Sum from $L_{out}$	<input type="text" value="0"/>	$R_{max}$	<input type="text" value="20"/> fm
Step	<input type="text" value="1"/> deg.	to $L_{max}$	<input type="text" value="40"/>	Integration step	<input type="text" value="0.1"/> fm
				$b_{max}$	<input type="text" value="9.55"/> fm
				$N_{traj}$	<input type="text" value="50"/>

**Calculate**

**Fitted parameters**  
 it is better to run first without fitting and use the option "dependence on..." (in the window of cross section) to find a sensitivity of the cross section to a given parameter

<i>Real part of Optical Potential</i>		<i>Imaginary part of Optical Potential</i>	
<input checked="" type="checkbox"/> Depth of Real Vol.	<input type="checkbox"/> Depth of Real Surf.	<input type="checkbox"/> Depth of Imag. Vol.	<input type="checkbox"/> Depth of Imag. Surf.
<input checked="" type="checkbox"/> Radius of Real Vol.	<input type="checkbox"/> Radius of Real Surf.	<input type="checkbox"/> Radius of Imag. Vol.	<input type="checkbox"/> Radius of Imag. Surf.
<input type="checkbox"/> Diffuseness of Re.Vol.	<input type="checkbox"/> Diffus. of Re.Surf.	<input type="checkbox"/> Diffuseness of Im.Vol.	<input type="checkbox"/> Diffus. of Im.Surf.
<i>Spin-Orbital Interaction</i>		<i>Coulomb Interaction</i>	
<input type="checkbox"/> Real part	<input type="checkbox"/> Radius	<input type="checkbox"/> Radius of the Coulomb Potential	
<input type="checkbox"/> Imaginary part	<input type="checkbox"/> Diffuseness	<i>Folding potential</i>	
		<input type="checkbox"/> $N_{Re}$	<input type="checkbox"/> $N_{Im}$

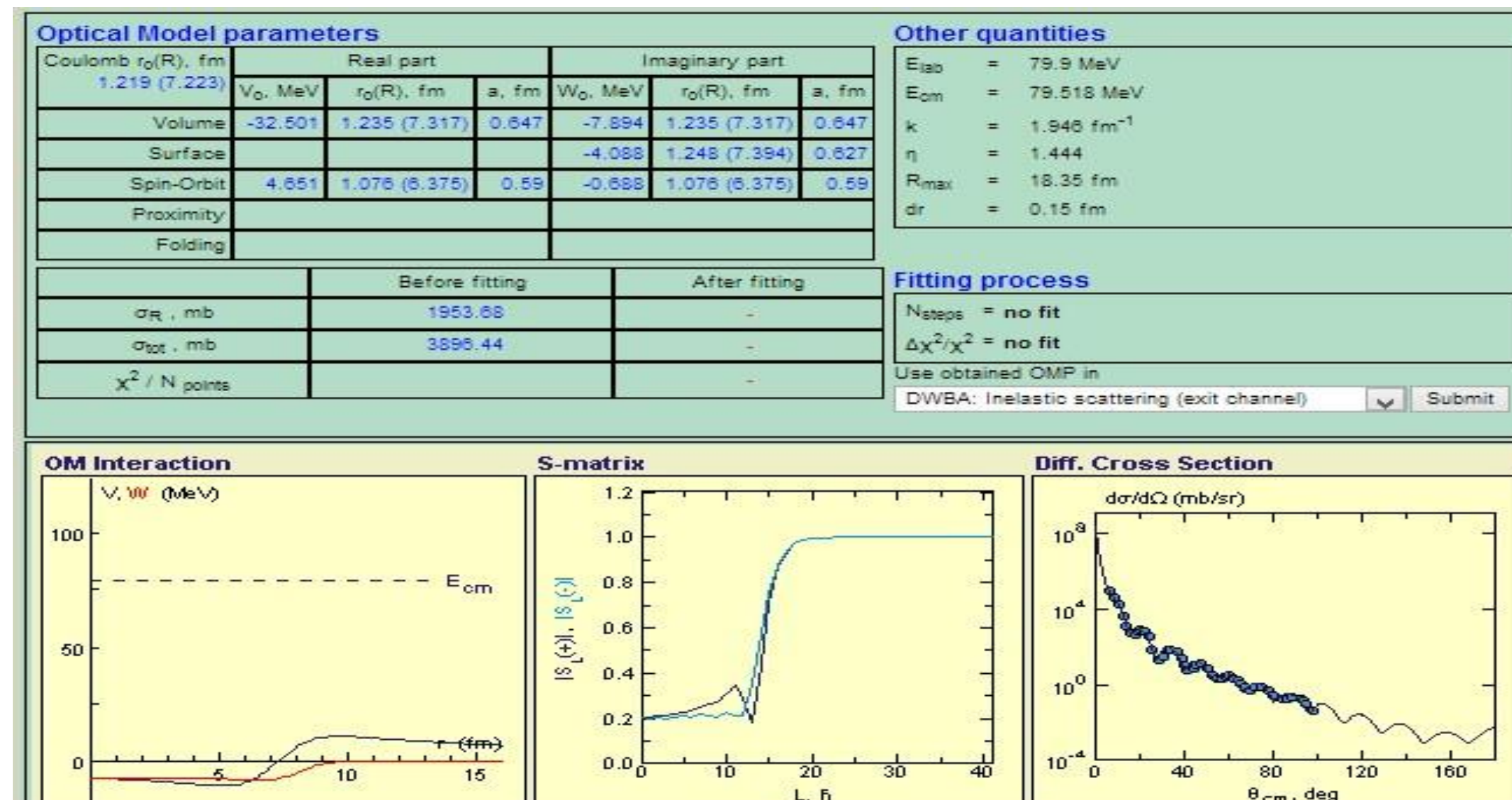
No fit    Maximal number of fit steps     <    >    Stop, when change is less than  %

## Optical Model calculation with NRV OM code

### Main steps of calculation:

#### Getting the results

- The differential cross section in absolute value or ratio to Rutherford.
- S-matrix
- The interaction potential



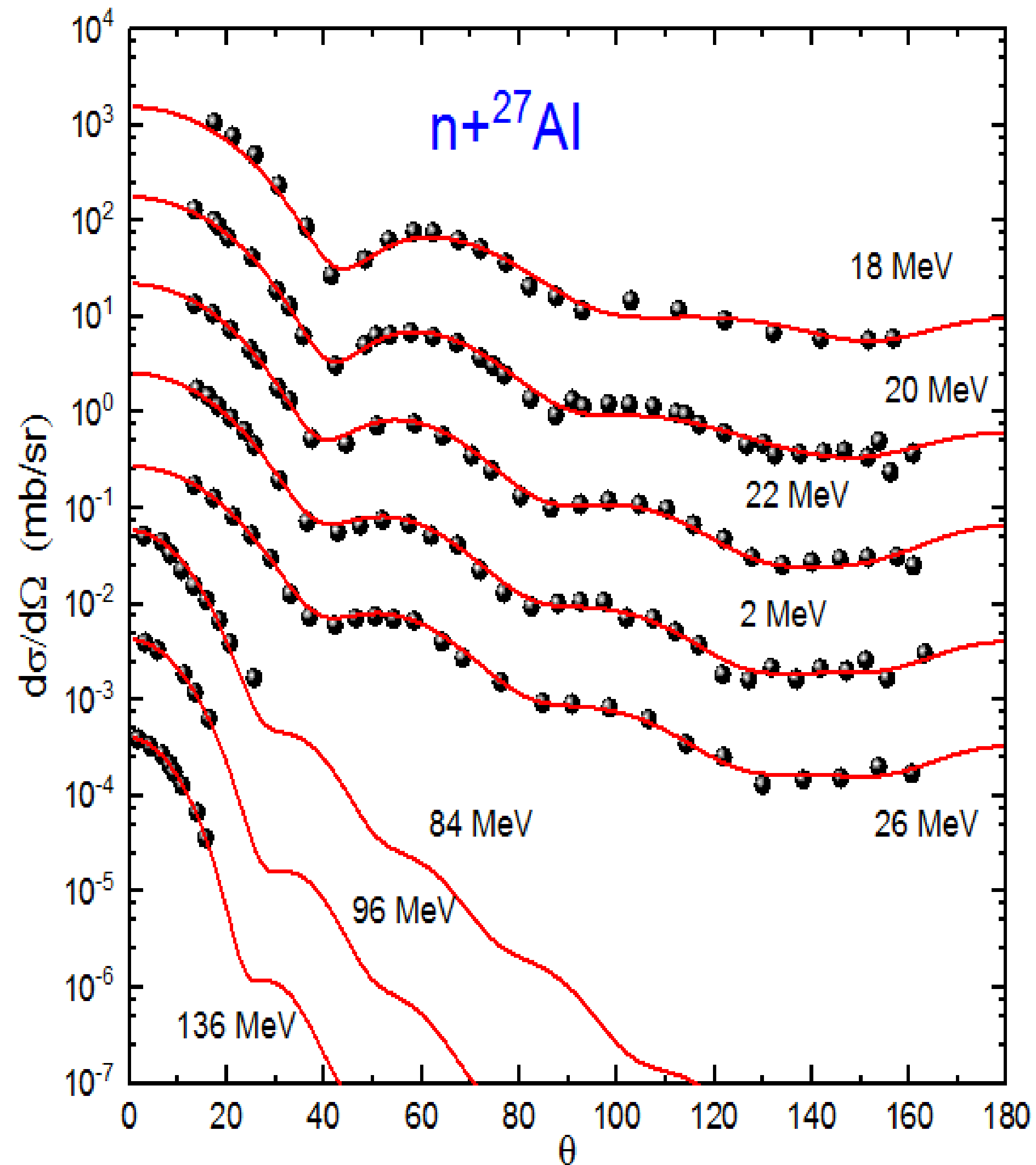
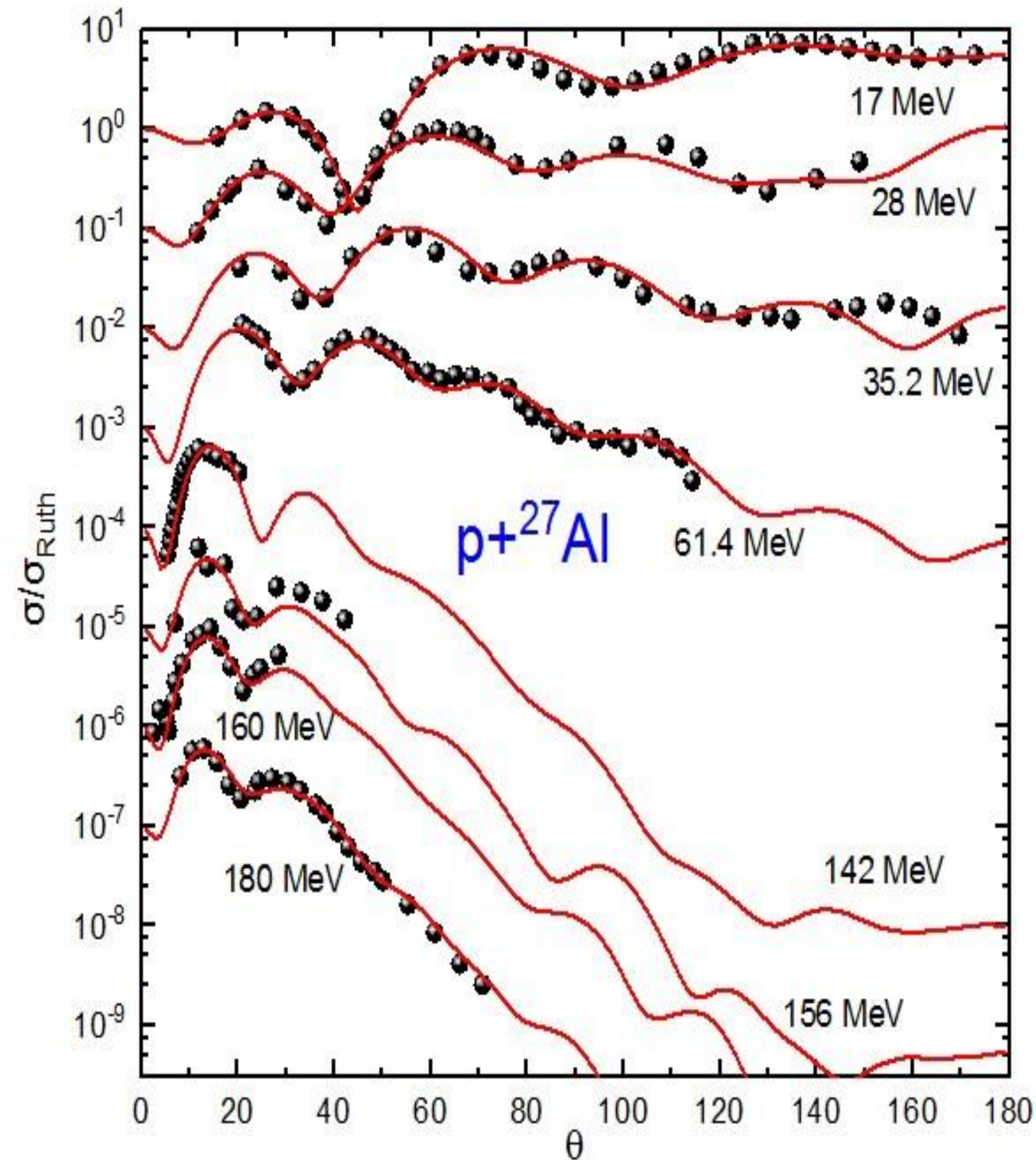


Figure 1:

## $n+^{27}\text{Al}$ elastic scattering:

The elastic scattering differential cross section of  $n+^{27}\text{Al}$  at different incident energies. The Woods-Saxon optical potential is used. The experimental data taken from [1].

[1] Koning, A., Delaroche, J.: Local and global nucleon optical models from 1 keV to 200 MeV. Nuclear Physics A 713(3-4), 231-310 (2003).



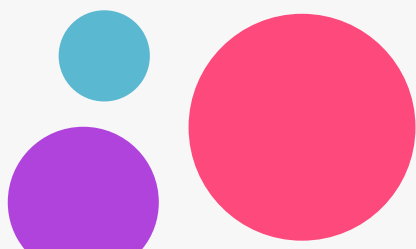
## Figure 2: p+<sup>27</sup>Al elastic scattering:

The elastic scattering differential cross section ratio to Rutherford of p+<sup>27</sup>Al at different incident energies. The Woods-Saxon optical potential is used. The experimental data taken from [1].

[1] Koning, A., Delaroche, J.: Local and global nucleon optical models from 1 keV to 200 MeV. Nuclear Physics A 713(3-4), 231-310 (2003).

Optical Model parameters for n,p+<sup>27</sup>Al. For all energies  $r_v=r_w=1.168$ ,  $a_v=a_w=0.674$ ,  $r_s=1.295$ ,  $a_s=0.533$ ,  $r_{so}=0.969$ , and  $a_{so}=0.59$ . The radius and diffuseness parameters are given in fm, while the potential depths in MeV.

Projectile	Target	E	V	Wv	Ws	Vso	Wso
n	<sup>27</sup> Al	18	-50.303	-1.579	-4.678	4.889	-0.08
		20	-50.485	-1.801	-5.218	5.026	-0.092
		22	-49.985	-2.03	-4.827	5.668	-0.106
		25	-43.717	-2.388	-4.461	6.493	-0.127
		26	-42.629	-2.15	-4.24	6.348	-0.134
		84	-38.73	-8.904	-2.197	4.154	-0.771
		96	-28.108	-9.76	-1.697	3.959	-0.921
		136	-24.987	-11.696	-0.713	3.374	-1.388
		P	<sup>27</sup> Al	17	-49.039	-1.472	-5.675
35.2	-44.466			-3.672	-4.822	5.049	-0.211
61.4	-36.404			-6.821	-2.333	5.116	-0.491
142	-18.637			-11.898	-0.626	3.294	-1.45
156	-10.361			-12.306	-2.16	3.947	-1.589
160	-13.635			-12.409	-3.327	6.685	-1.626
180	-4.197			-12.845	-2.821	5.519	-1.798





# The energy dependence of the OP:-

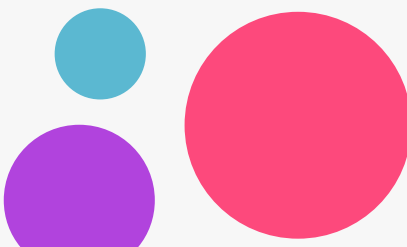
By studying the elastic scattering of protons with different incident energies and different targets, we got the following approximation relations for the real part of the optical potential.

$$V = -65.75 - 1.837 \left( \frac{N-Z}{A} \right) - 14.41 E_{lab} + 15.57 E_{lab}^{0.988}.$$

$$r_v = 1.11 + 0.0025 A - 1.95 \times 10^{-5} A^2 + 5.59 \times 10^{-8} A^3.$$

$$a_v = 0.563 + 0.0025 A - 9.8 \times 10^{-6} A^2.$$

These relations produce got fitting with the experimental data, as shown in the following figures:



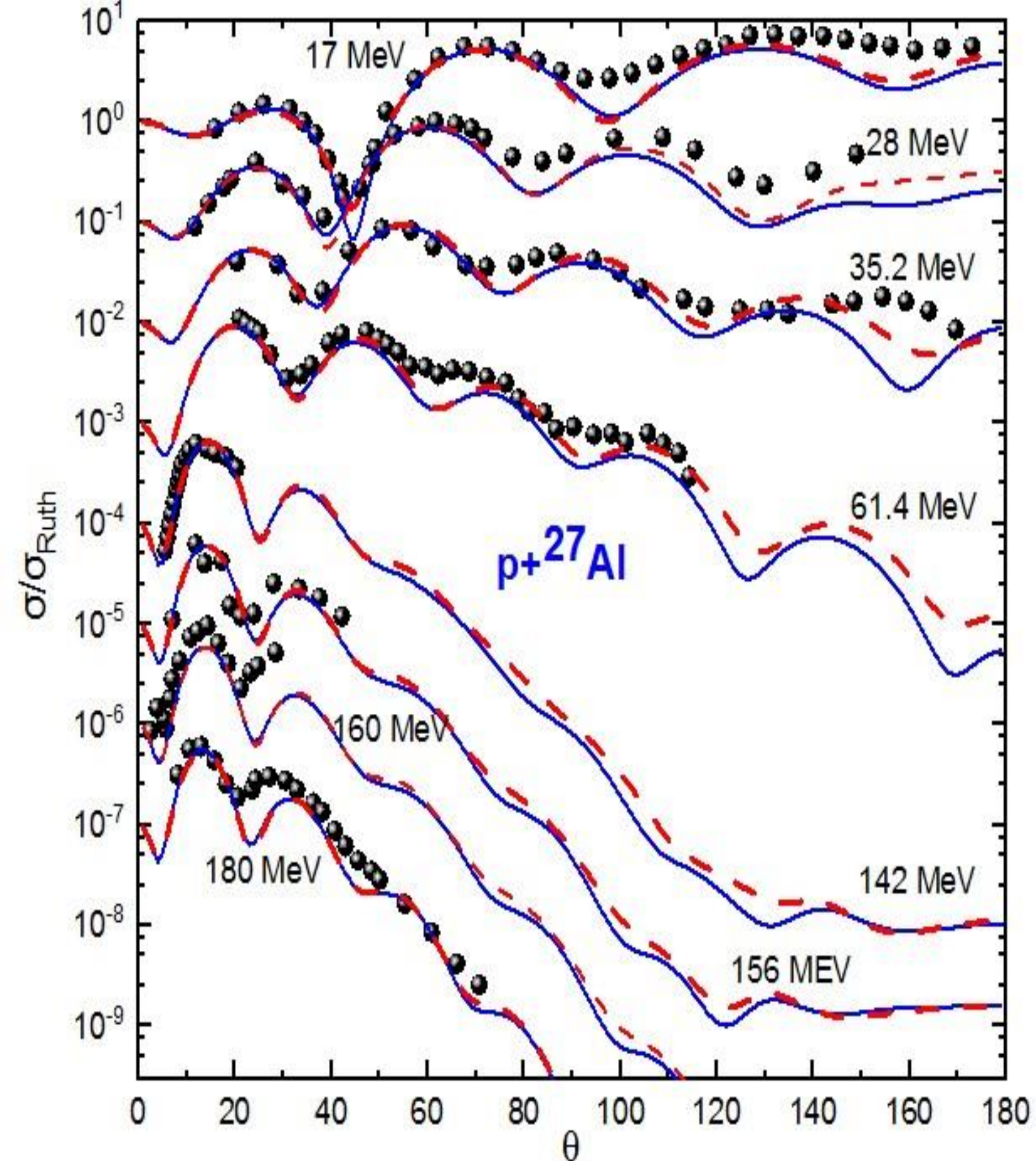
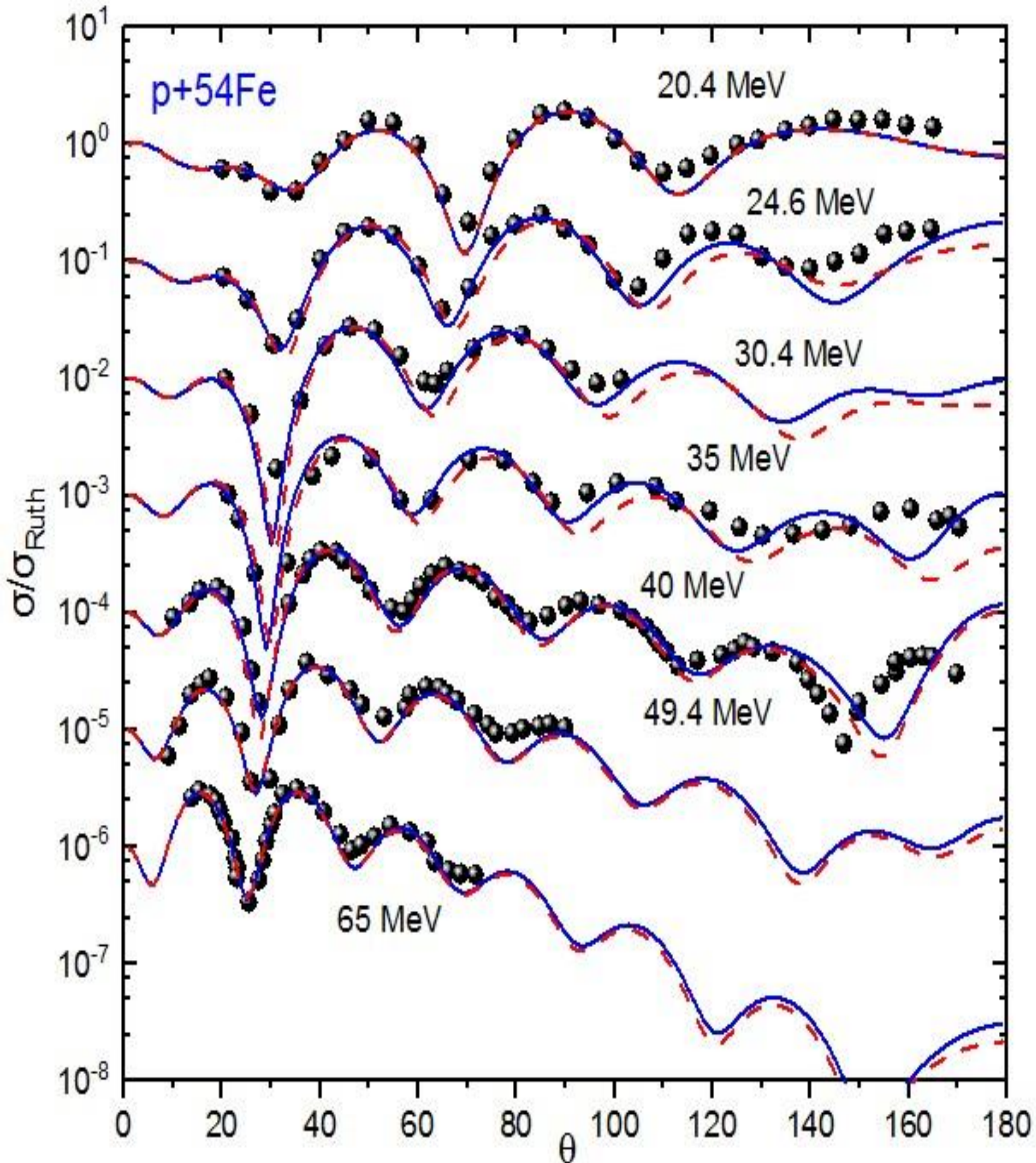


Figure 3:

## $p+^{27}\text{Al}$ elastic scattering:

The elastic scattering differential cross section ratio to Rutherford of  $p+^{27}\text{Al}$  at different incident energies. The solid lines show OM calculation results using Koning and Delaroche global optical potential [1] while the dashed lines show our results.

[1] Koning, A., Delaroche, J.: Local and global nucleon optical models from 1 keV to 200 MeV. Nuclear Physics A 713(3-4), 231-310 (2003).



## Figure 4: $p+^{54}Fe$ elastic scattering:

The elastic scattering differential cross section ratio to Rutherford of  $p+^{54}Fe$  at different incident energies. The solid and the dashed lines same as Fig. 3.

[1] Koning, A., Delaroche, J.: Local and global nucleon optical models from 1 keV to 200 MeV. Nuclear Physics A 713(3-4), 231-310 (2003).

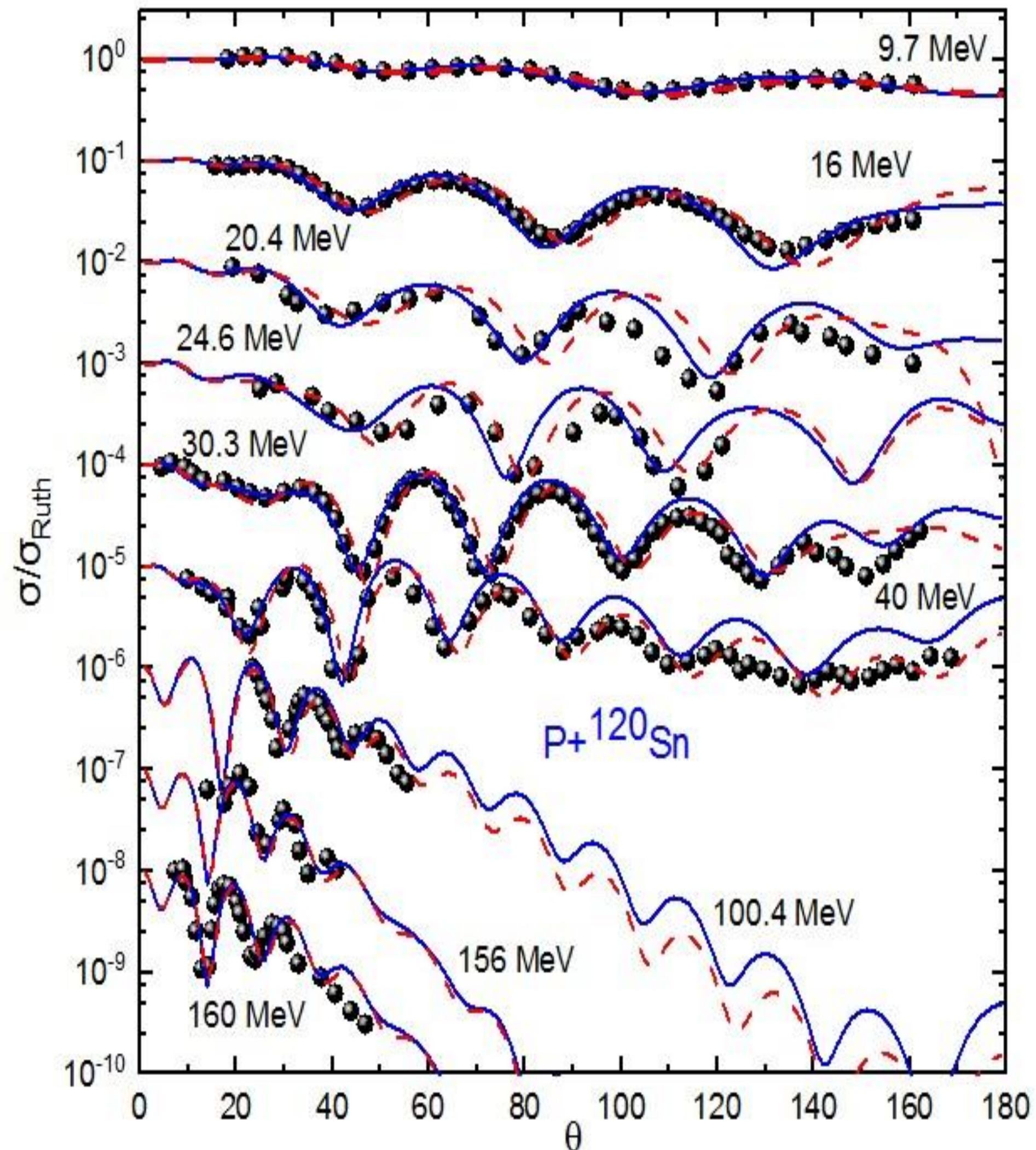


Figure 1:

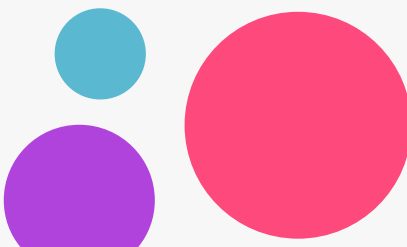
## $p+^{120}\text{Sn}$ elastic scattering:

The elastic scattering differential cross section ratio to Rutherford of  $p+^{120}\text{Sn}$  at different incident energies. The solid and the dashed lines same as Fig. 3.

[1] Koning, A., Delaroche, J.: Local and global nucleon optical models from 1 keV to 200 MeV. Nuclear Physics A 713(3-4), 231-310 (2003).

# Conclusions

- We studied the elastic scattering theory and the Optical Model.
- We derived the expressions for
  - partial wave expansion of a plane wave,
  - a relation between the elastic cross section and phase shifts
  - a relation between the scattering amplitude and the phase shifts
- We applied the NRV OM code to study elastic scattering of proton and neutron off different targets at different energies
- We obtained approximate relationships for the real part of the optical potential that give good fitting with the experimental data.
- Good agreement between calculation results and experimental data was achieved.



---



Thank you for your attention

Any questions?

