Study of heavy-ion elastic scattering within classical and quantum optical model

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Academy of Scientific Research & Technology

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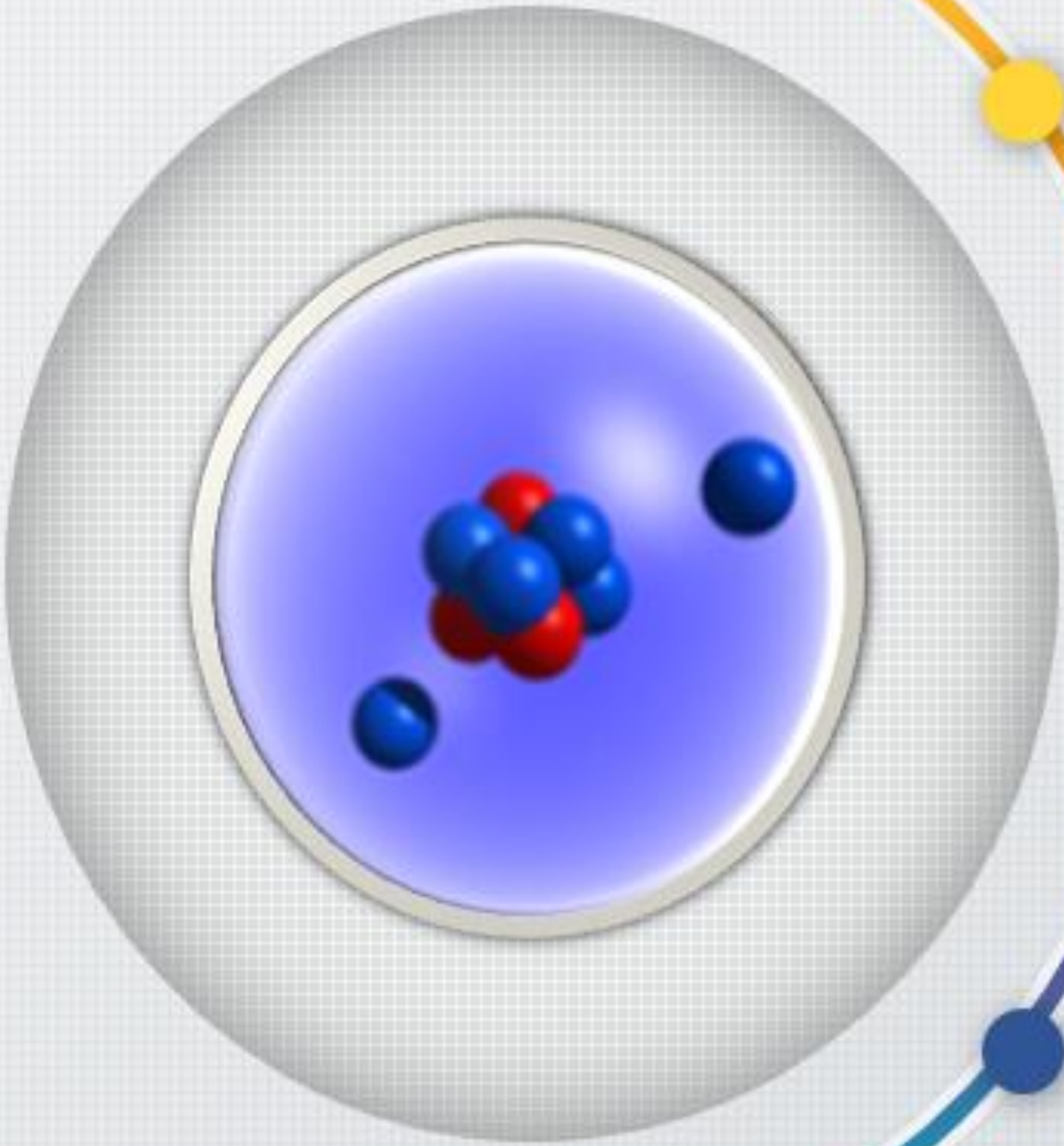


Aim of the work

1- The project aims to study of the behavior of the elastic scattering differential cross section for different energies and different nucleon-target combinations

2- Derivation of expressions for:

a- partial wave expansion of a plane wave.

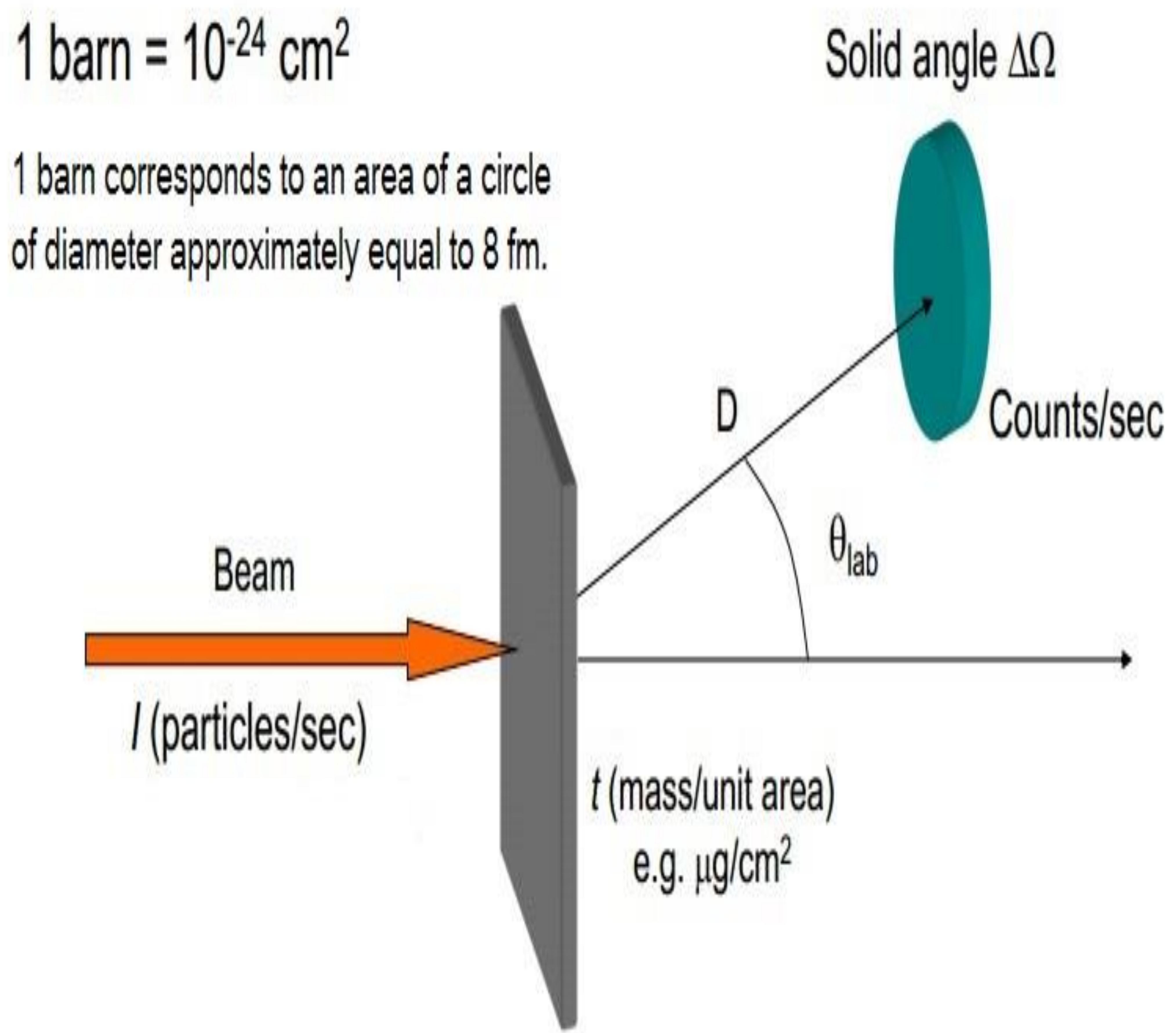
b- relation between the elastic cross section and phase shifts.

c- relation between the scattering amplitude and the phase shifts

3- Digitizing experimental data from the graphs of articles using GSYS program and Importing data in the NRV project for low energy codes

4- Search for the optical potential parameters which produce best fit with the experimental data. And finally Interpreting the results

Intro:



The nuclear reactions:

A large fraction of our knowledge about the internal structure of nuclei is came from the nuclear reactions.

$$N_\alpha(\Omega, \Delta\Omega)\ \alpha\ (\Delta\Omega . n . j)$$

$$N_\alpha(\Omega, \Delta\Omega) = (\Delta\Omega . n . j).(\text{proportionality constan})$$

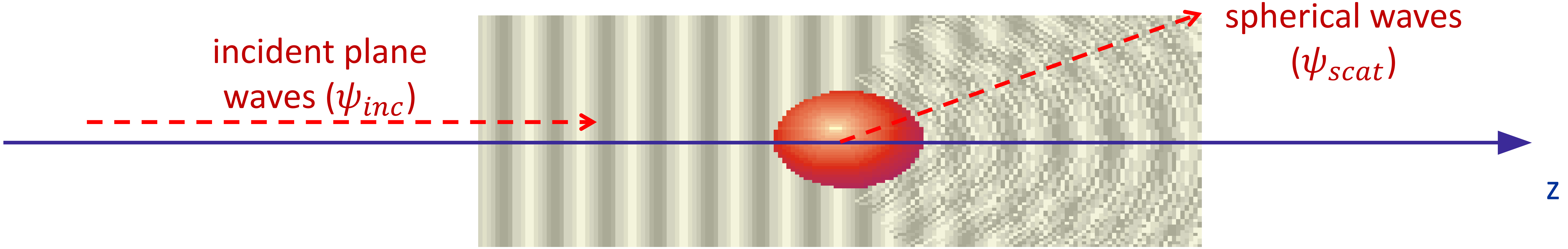
$$N_\alpha(\Omega, \Delta\Omega) = (\Delta\Omega \cdot n \cdot J)\ d\sigma_\alpha(\Omega)/d\Omega$$

$$\frac{d\sigma_\alpha(\Omega)}{d\Omega} = \frac{N_\alpha(\Omega, \Delta\Omega)}{\Delta\Omega \cdot n \cdot J},$$ → Differential Cross Section

$$\sigma_\alpha = \int d\Omega \left[\frac{d\sigma_\alpha(\Omega)}{d\Omega}\right]$$ → The Reaction Cross Section

In the center of mass frame, the problem of scattering between two particles (p and t) is reduced to the scattering of a particle of mass (μ) by a finite range central potential $V(\mathrm{r})$.

$$\left[-\frac{\hbar^2}{2\mu}\left(\nabla^2+k^2\right)+V(\vec{r})\right]\psi(\vec{r})=0 \qquad (1)$$



The solution of this equation is:

$$\psi(\vec{r}) = \psi_{inc}(\vec{r}) + \psi_{sca}(\vec{r}),$$

where

$$\left.\begin{aligned}\psi_{inc}(\vec{r}) &= Ae^{ik_0 z}.\\ \psi_{sca}(\vec{r}) &= A\, f(\theta,\phi)\frac{e^{ikr}}{r}.\end{aligned}\right\} \quad ,\mathrm{r} \gg a$$

- The most general solution of the Schrödinger equation (1) is

$$\psi(\vec{r}) = \frac{u_{El}(r)}{r} Y_{lm}(\theta, \phi)$$

- When the potential is central, the scattered wave function don't depend on (ϕ) and hence m=0. Thus:

$$\psi(\vec{r}) = \frac{u_{El}(r)}{r} Y_{l0}(\theta)$$

- Where $u_{El}(r)$ is the solution of the radial wave function, which is expressed as the following in case of free particles ($V(r) = 0$):

$$\left(-\frac{d^2}{dr^2} + \frac{l(l+1)}{r^2}\right) u_{El}(r) = k^2 u_{El}(r)$$

- The general solution of this equation is given by a linear combination of the spherical Bessel and Neumann functions

$$u_{El} = A_l \rho\, j_l(\rho) + B_l \rho\, n_l(\rho), \qquad \rho = kr$$

- Thus one can generate plane waves from a linear combination of the spherical Bessel and Neumann functions:

$$e^{i\vec{k}.\vec{r}} = e^{ikrcos\theta} = \sum_l a_l \big(A_l\, j_l(\rho) + B_l\, n_l(\rho)\big) Y_{l0}(\theta)$$

➡ The most general solution of the Schrödinger equation (1) is

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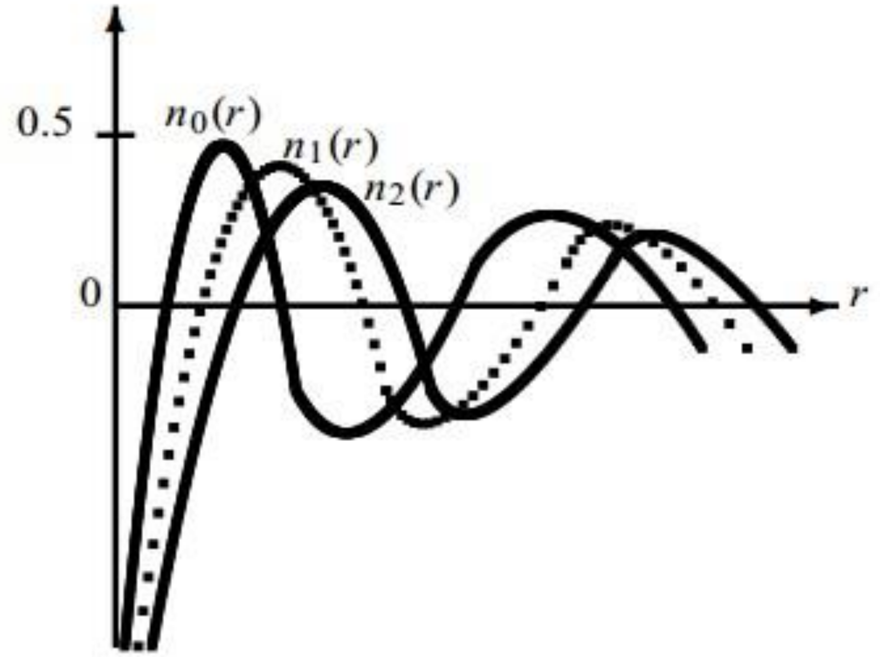


A substitution of a_l and $j_l(\rho)$ into the previous equation:

$$e^{ikr} \cong \frac{\sqrt{4\pi}}{k}\sum_l \sqrt{2l+1}\, i^l Y_{l0}\, \frac{1}{2i}\left(\frac{e^{i\left(kr-\frac{l\pi}{2}\right)}}{r} - \frac{e^{-i\left(kr-\frac{l\pi}{2}\right)}}{r}\right), \mathrm{r} \gg a \qquad (1)$$

Thus the scattered wave function is going to be

$$\psi(\vec{r}) = \psi_{inc}(\vec{r}) + \psi_{sca}(\vec{r})$$

$$= \frac{\sqrt{4\pi}}{k}\sum_l \sqrt{2l+1}\, i^l Y_{l0}\, \frac{1}{2i}\left(\frac{e^{i\left(kr-\frac{l\pi}{2}\right)}}{r} - \frac{e^{-i\left(kr-\frac{l\pi}{2}\right)}}{r}\right) + f_k(\theta)\frac{e^{ikr}}{r} \quad r \gg a \qquad (2)$$

The right hand side of equation (1) has to take the folloeing form:

$$\psi(\vec{r}) = \frac{\sqrt{4\pi}}{k}\sum_l \sqrt{2l+1}\, i^l Y_{l0}\, \frac{1}{2i}\left(\frac{e^{-i\left(kr-\frac{l\pi}{2}\right)+2i\delta_l}}{r} - \frac{e^{-i\left(kr-\frac{l\pi}{2}\right)}}{r}\right) \qquad (3)$$

Where δ_l is the phase shifts. By comparison (1) and (2) one can fined:

$$f_k(\theta) = \frac{\sqrt{4\pi}}{k}\sum_l \sqrt{2l+1}\, Y_{l0}(\theta) e^{i\delta_l} \sin(\delta_l)\,.$$

Finally →

$$\frac{d\sigma}{d\Omega} = \left|f_k(\theta)\right|^2$$

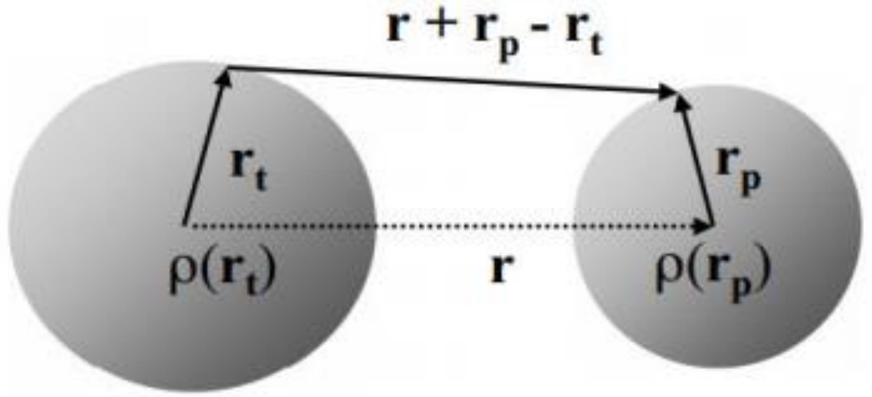
The Optical Poential:

There are basically two ways to describe the optical potential:

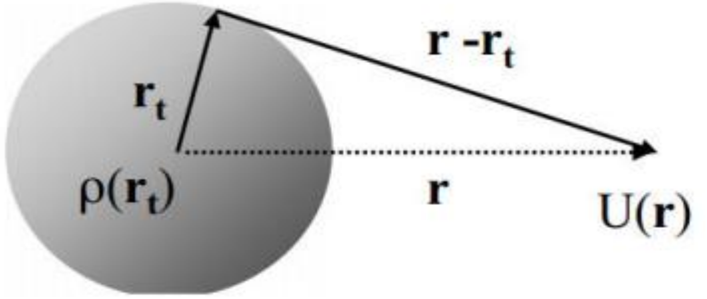
(1) Phenomenological Optical potential

$$
\begin{aligned}
V(r) = &-V f_1(r) && \text{a real volume term,}\\
&+iW_v f_2(r) && \text{an imaginary volume term,}\\
&-iW_d \tfrac{d}{dr} f_3(r) && \text{an imaginary surface term,}\\
&-V_{s.o} \tfrac{b}{r}\tfrac{d}{dr} f_3(r)\vec{\ell}.\vec{\sigma} && \text{a real spin} - \text{orbit term,}\\
&-iW_{s.o} \tfrac{b}{r}\tfrac{d}{dr} f_4(r)\vec{\ell}.\vec{\sigma} && \text{an imaginary spin} - \text{orbit term}\\
&+V_C(r) && \text{and Coulomb term.}
\end{aligned}
$$

(2) Microscopic Folding potential approach



$$V_{DF}(r) = \int\int V_{NN}(\mathbf{r} + \mathbf{r}_P - \mathbf{r}_T)\,\rho_T(\mathbf{r}_T)\rho_P(\mathbf{r}_P)\,d\mathbf{r}_T d\mathbf{r}_P,$$



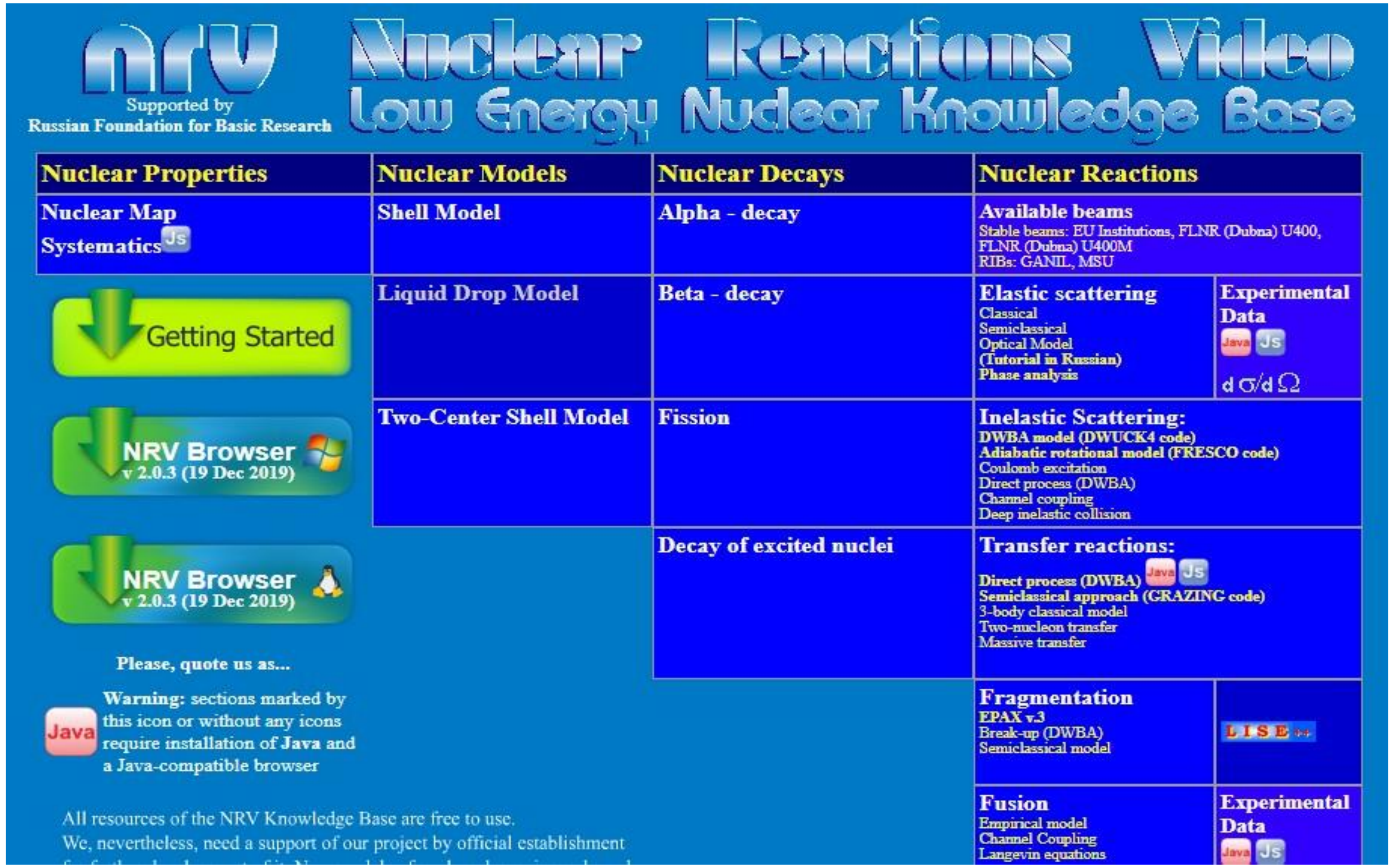
$$V_{SF}(r) = \int V_{NN}(\mathbf{r} - \mathbf{r}_T)\,\rho_T(\mathbf{r}_T)\,d\mathbf{r}_T.$$

Practical part

NRV knowledge base   http://nrv.jinr.ru

The NRV web knowledge base is a unique interactive research system:

- Allows to run complicated computational codes
- Works NRV browser, which can be downloaded free
- Has graphical interface for preparation of input parameters and analysis of output results
- Combines computational codes with experimental databases on properties of nuclei and nuclear reactions
- Contains detailed description of models



Practical part

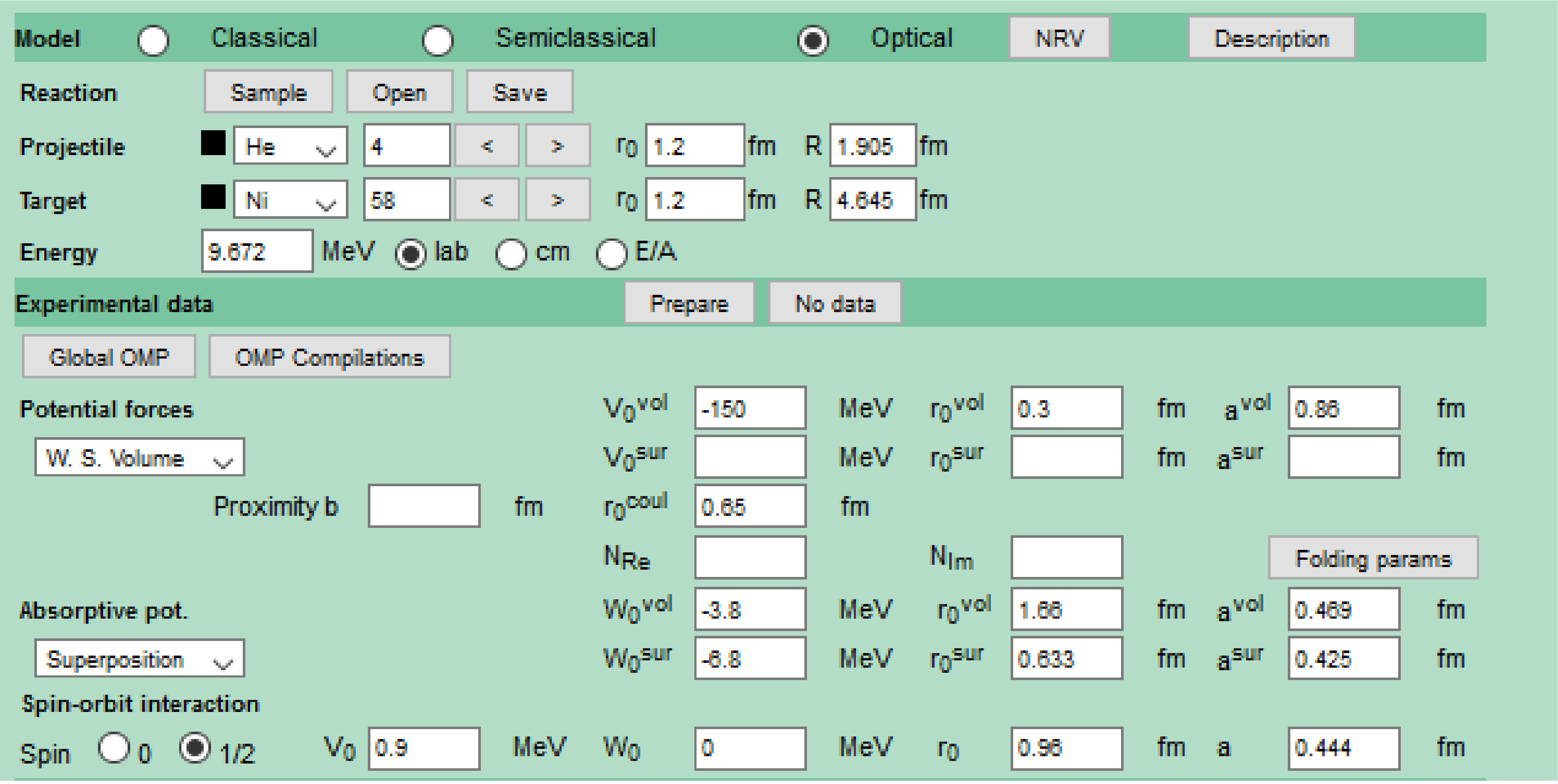
http://nrv.jinr.ru

Optical Model calculation with NRV OM code

Main steps of calculation:

Physical

- Set projectile and target parameters (mass, spin, etc)
- Set the incident energy
- Set the parameters of the OM potential

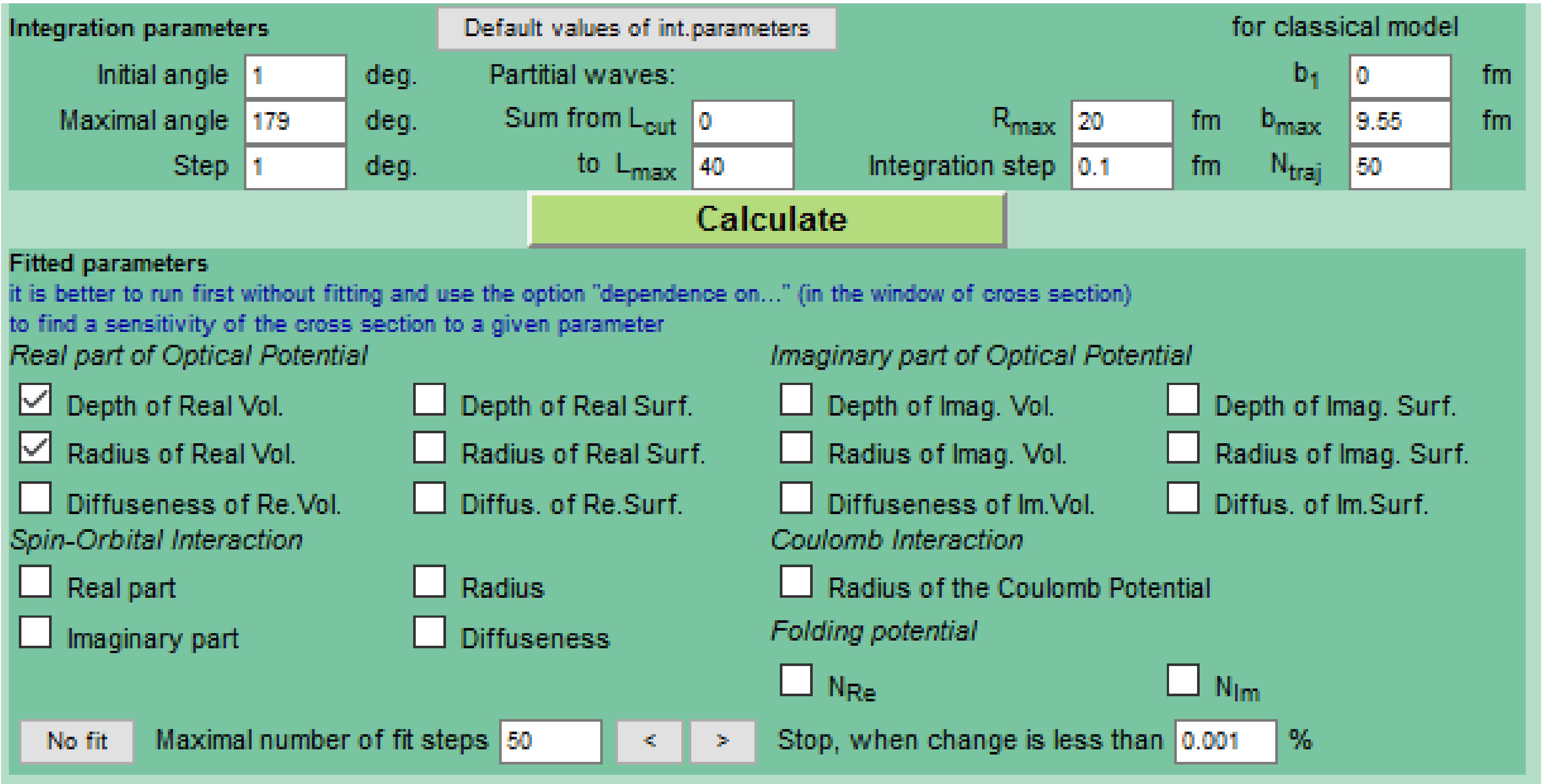


Optical Model calculation with NRV OM code

Main steps of calculation:

Numerical

- Set the radial step for integration
- Set the maximum radius *R* for integration
- Set the maximum angular momentum *L*

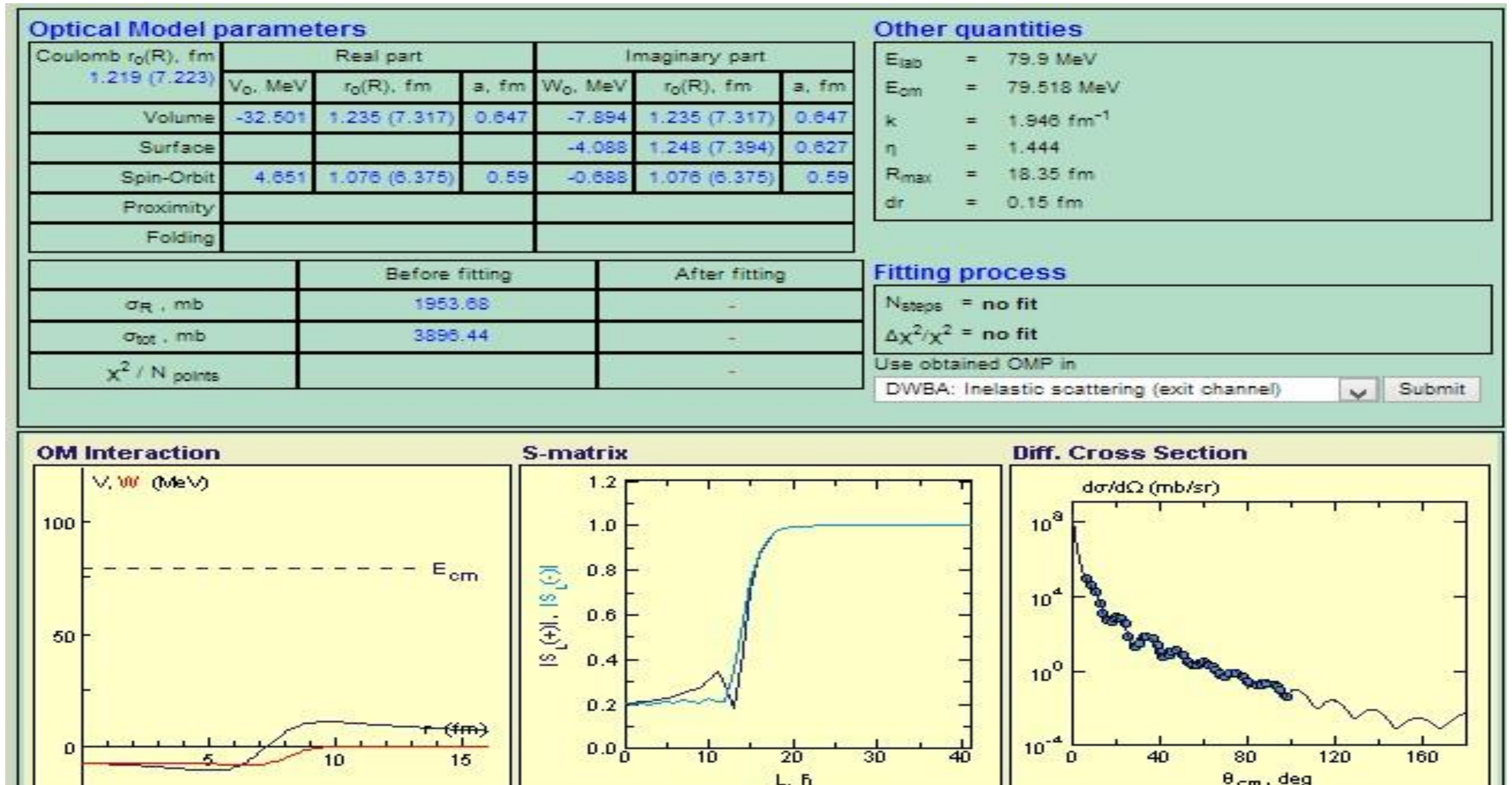


Optical Model calculation with NRV OM code

Main steps of calculation:

Getting the results

- The differential cross section in absolute value or ratio to Rutherford.
- S-matrix
- The interaction potential



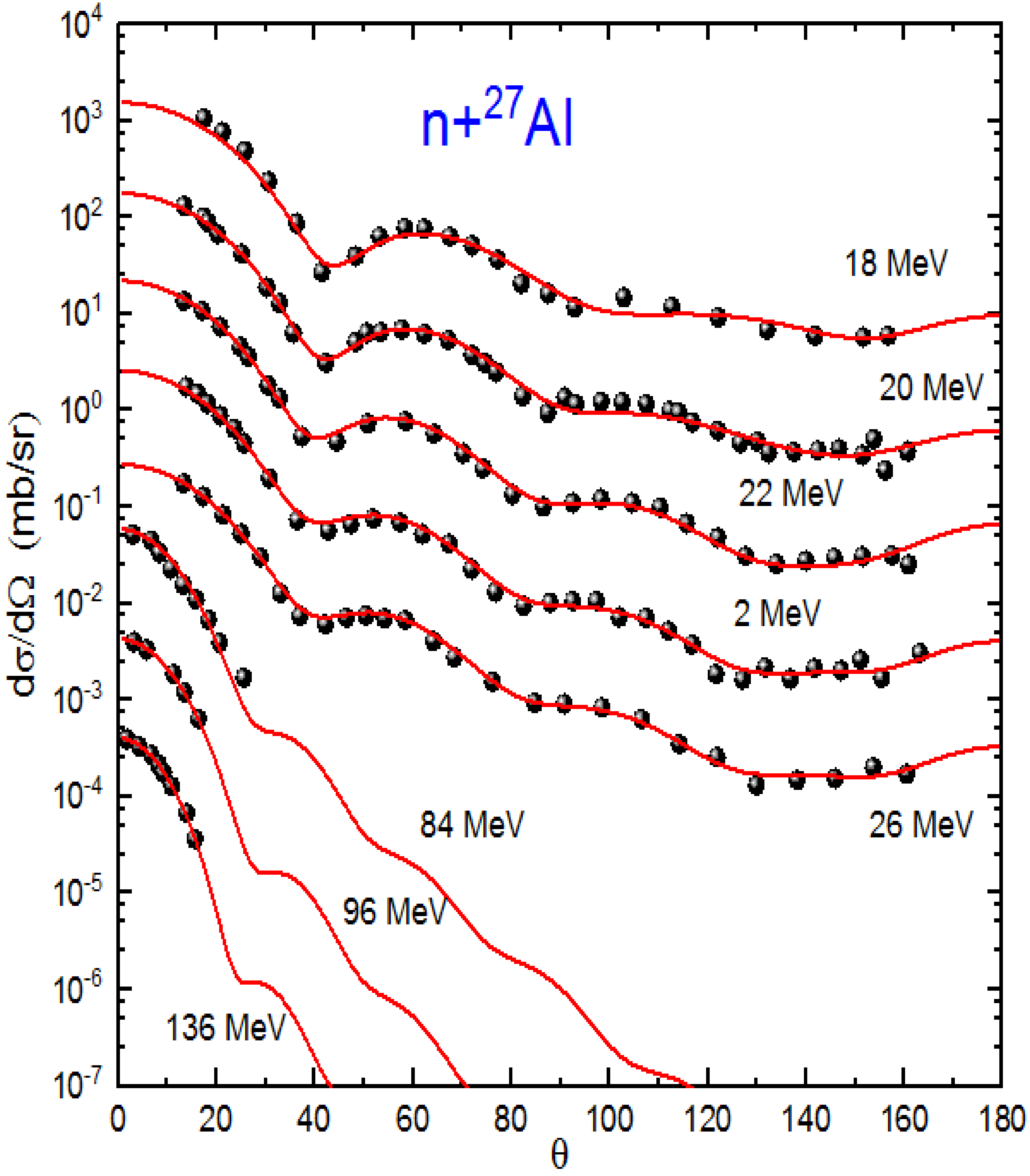


Figure 1:

$n+^{27}Al$ elastic scattering:

The elastic scattering differential cross section of $n+^{27}Al$ at different incident energies. The Woods-Saxon optical potential is used. The experimental data taken from [1].

[1] Koning, A., Delaroche, J.: Local and global nucleon optical models from 1 keV to 200 MeV. Nuclear Physics A 713(3-4), 231-310 (2003).

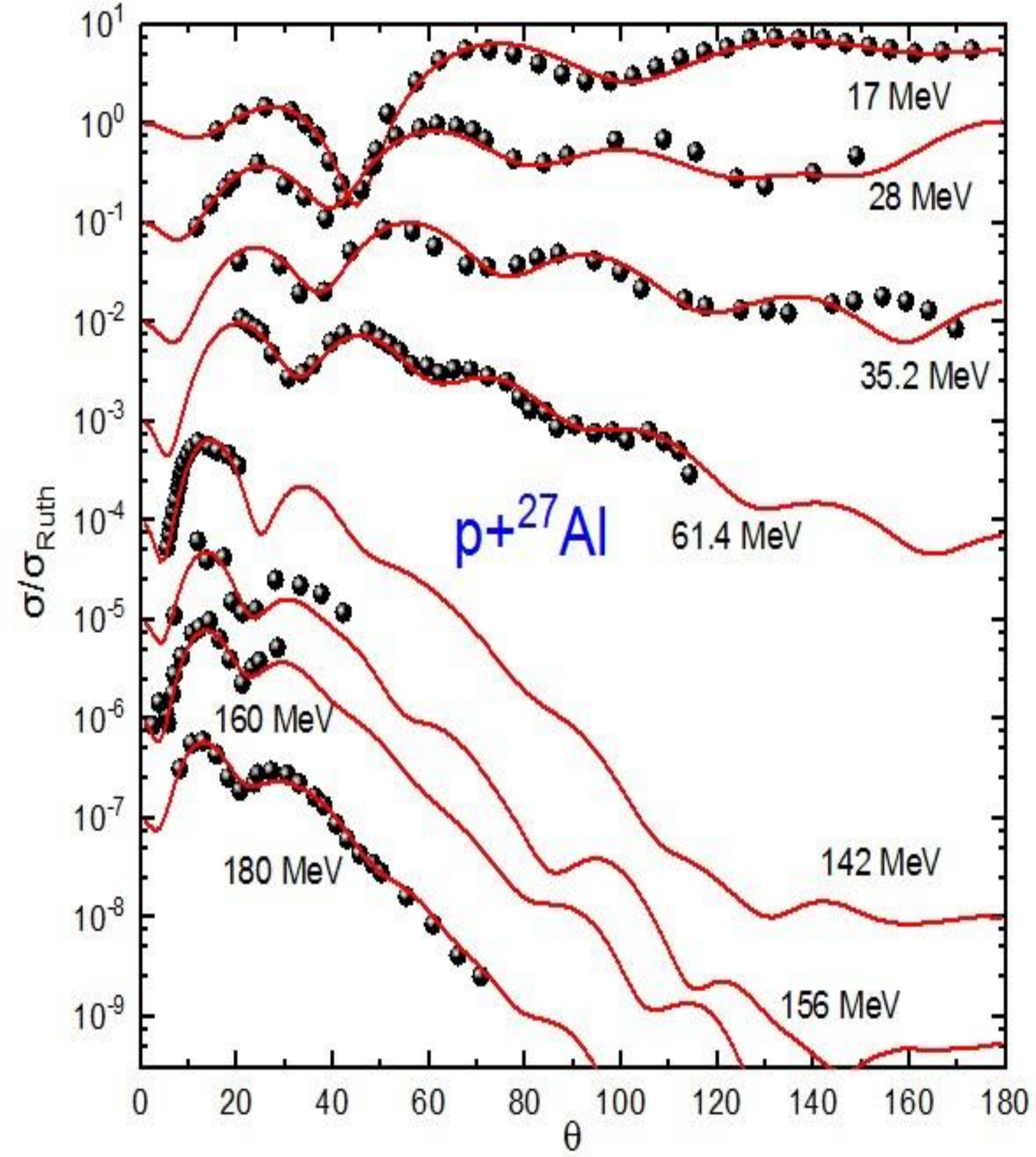


Figure 2:
p+^{27}Al elastic scattering:

The elastic scattering differential cross section ratio to Rutherford of p+^{27}Al at different incident energies. The Woods-Saxon optical potential is used. The experimental data taken from [1].

[1] Koning, A., Delaroche, J.: Local and global nucleon optical models from 1 keV to 200 MeV. Nuclear Physics A 713(3-4), 231-310 (2003).

Optical Model parameters for n,p+^{27}Al. For all energies $r_v=r_w=1.168$, $a_v=a_w=0.674$, $r_s=1.295$, $a_s=0.533$, $r_{so}=0.969$, and $a_{so}=0.59$. The radius and diffuseness parameters are given in fm, while the potential depths in MeV.

Projectile	Target	E	V	Wv	Ws	Vso	Wso
n	^{27}Al	18	-50.303	-1.579	-4.678	4.889	-0.08
		20	-50.485	-1.801	-5.218	5.026	-0.092
		22	-49.985	-2.03	-4.827	5.668	-0.106
		25	-43.717	-2.388	-4.461	6.493	-0.127
		26	-42.629	-2.15	-4.24	6.348	-0.134
		84	-38.73	-8.904	-2.197	4.154	-0.771
		96	-28.108	-9.76	-1.697	3.959	-0.921
		136	-24.987	-11.696	-0.713	3.374	-1.388
P	^{27}Al	17	-49.039	-1.472	-5.675	6.953	-0.074
		35.2	-44.466	-3.672	-4.822	5.049	-0.211
		61.4	-36.404	-6.821	-2.333	5.116	-0.491
		142	-18.637	-11.898	-0.626	3.294	-1.45
		156	-10.361	-12.306	-2.16	3.947	-1.589
		160	-13.635	-12.409	-3.327	6.685	-1.626
		180	-4.197	-12.845	-2.821	5.519	-1.798

The energy dependence of the OP:-

By studying the elastic scattering of protons with different incident energies and different targets, we got the following approximation relations for the real part of the optical potential.

$$V = -65.75 - 1.837\left(\frac{N-Z}{A}\right) - 14.41\, E_{lab} + 15.57\, E_{lab}^{0.988}.$$

$$r_v = 1.11 + 0.0025\, A - 1.95 \times 10^{-5}\, A^2 + 5.59 \times 10^{-8}\, A^3.$$

$$a_v = 0.563 + 0.0025\, A - 9.8 \times 10^{-6}\, A^2.$$

These relations produce got fitting with the experimental data, as shown in the following figures:

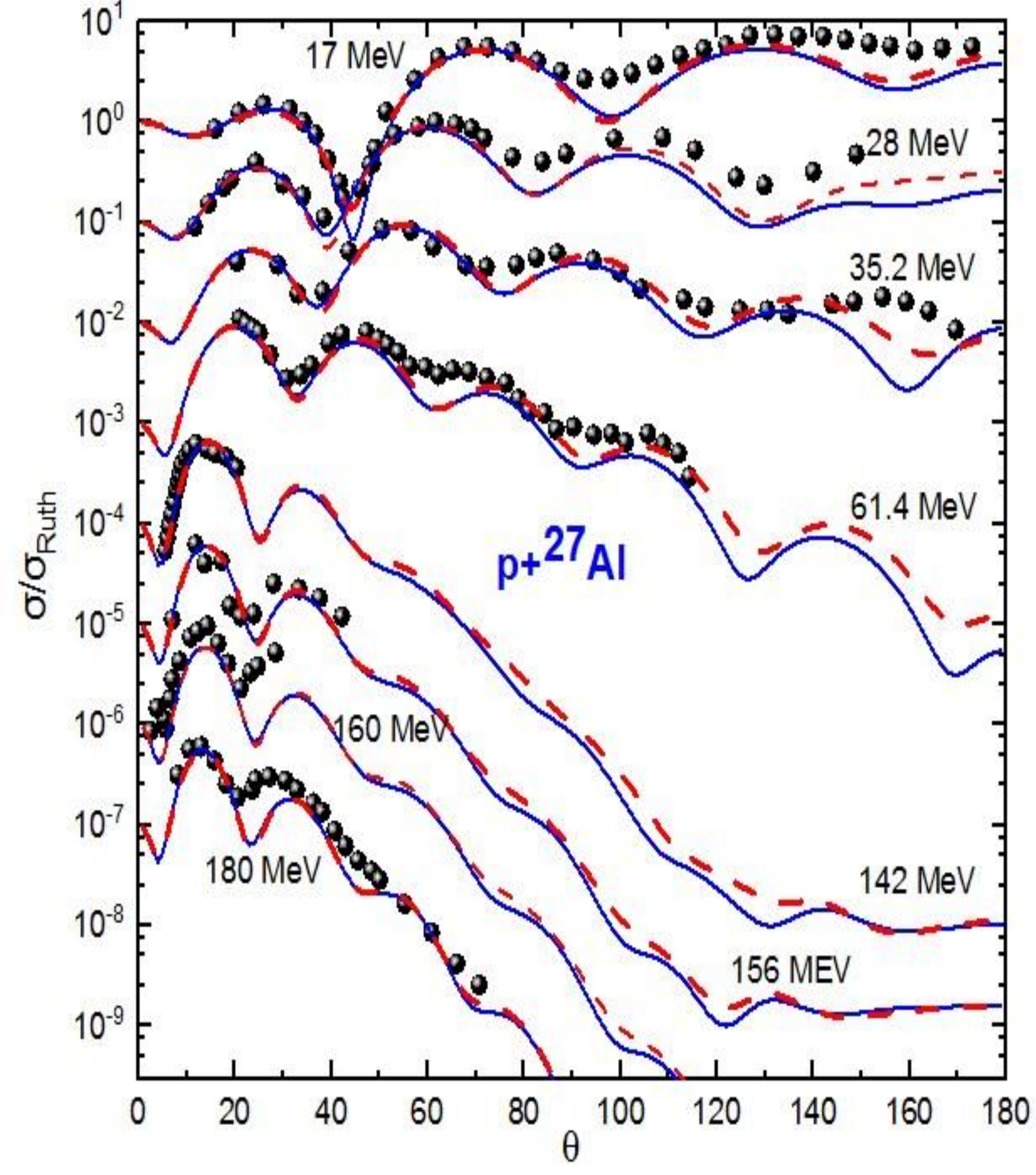


Figure 3:

p+^{27}Al elastic scattering:

The elastic scattering differential cross section ratio to Rutherford of p+^{27}Al at different incident energies. The solid lines show OM calculation results using Koning and Delaroche global optical potential [1] while the dashed lines show our results.

[1] Koning, A., Delaroche, J.: Local and global nucleon optical models from 1 keV to 200 MeV. Nuclear Physics A 713(3-4), 231-310 (2003).

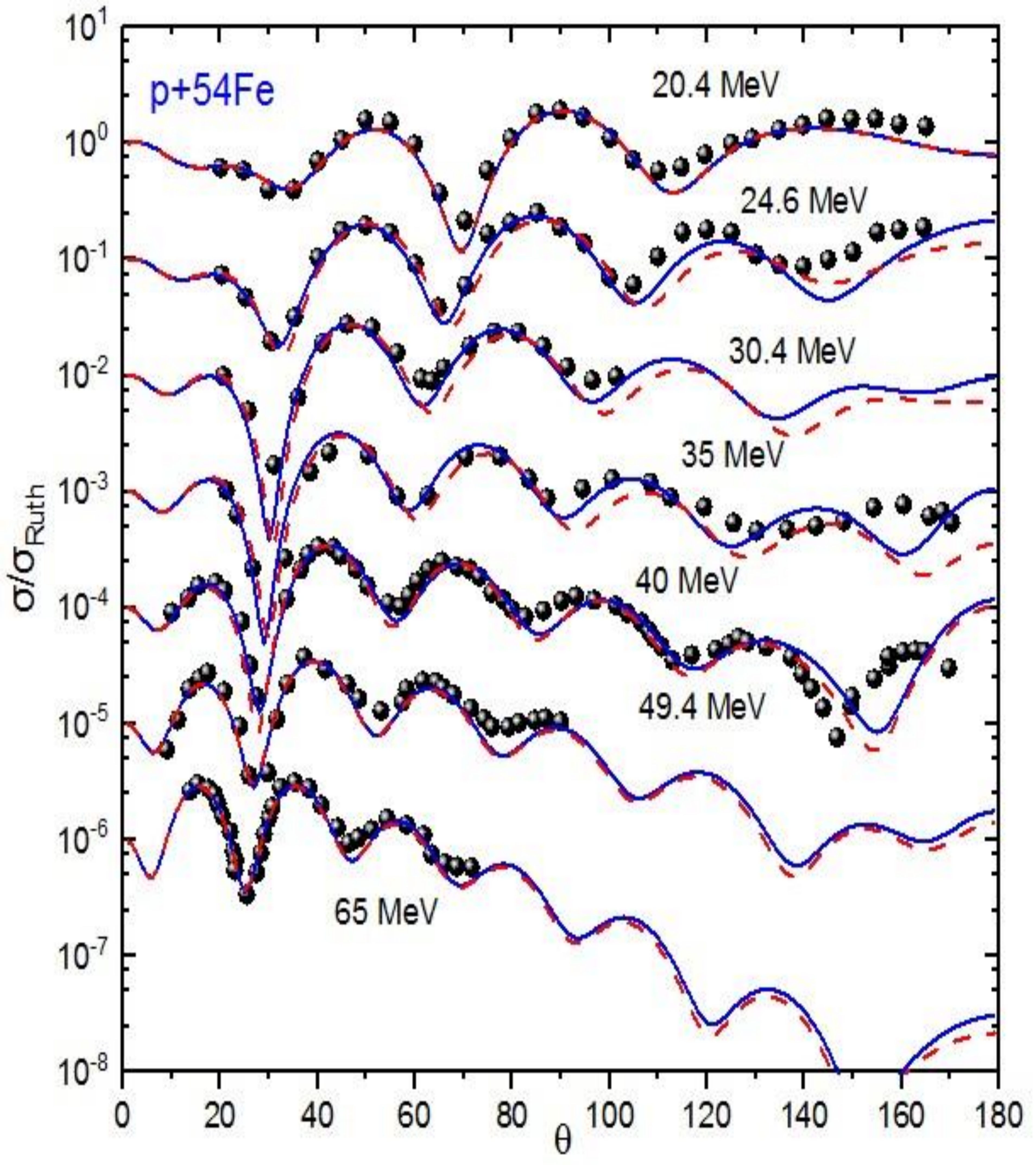


Figure 4:

$p+^{54}Fe$ elastic scattering:

The elastic scattering differential cross section ratio to Rutherford of $p+^{54}Fe$ at different incident energies. The solid and the dashed lines same as Fig. 3.

[1] Koning, A., Delaroche, J.: Local and global nucleon optical models from 1 keV to 200 MeV. Nuclear Physics A 713(3-4), 231-310 (2003).

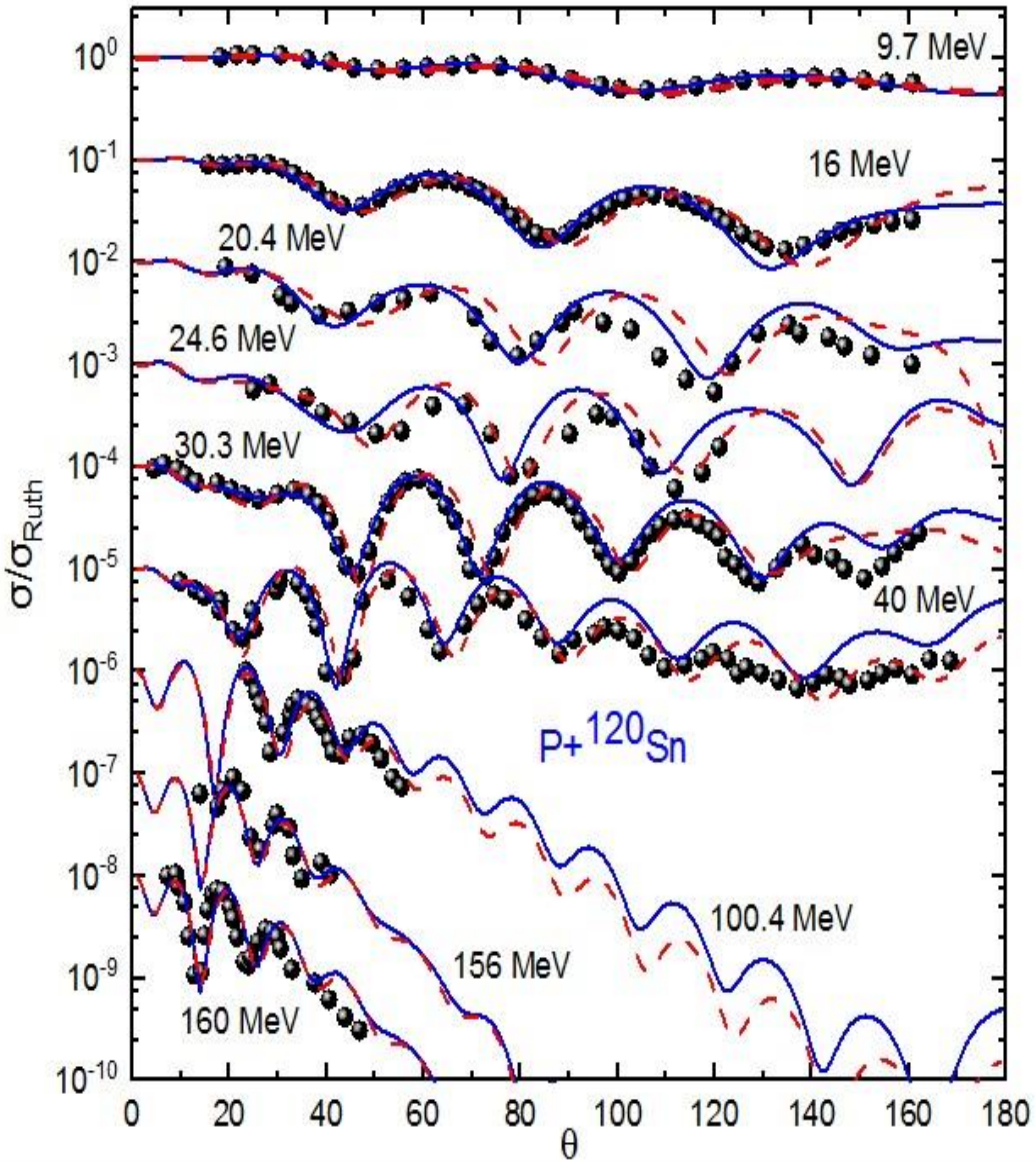


Figure 1:

p+^{120}Sn elastic scattering:

The elastic scattering differential cross section ratio to Rutherford of p+^{120}Sn at different incident energies. The solid and the dashed lines same as Fig. 3.

[1] Koning, A., Delaroche, J.: Local and global nucleon optical models from 1 keV to 200 MeV. Nuclear Physics A 713(3-4), 231-310 (2003).

Conclusions

- We studied the elastic scattering theory and the Optical Model.
- We derived the expressions for
  - partial wave expansion of a plane wave,
  - a relation between the elastic cross section and phase shifts
  - a relation between the scattering amplitude and the phase shifts
- We applied the NRV OM code to study elastic scattering of proton and neutron off different targets at different energies
- We obtained approximate relationships for the real part of the optical potential that give good fitting with the experimental data.
- Good agreement between calculation results and experimental data was achieved.

Thank you for your attention

Any questions?