

**Title**

***Numerical Estimates of the Mellin-Barnes Integrals Based on the Contour of the Stationary Phase***

**Description of the project**

The important problem in high-energy physics is effective calculation of the Mellin-Barnes (MB) integrals – a family of integrals in the complex plane whose integrand is given by the ratio of products of Gamma functions [1]. In the quantum theory the list of the tasks solved by using the MB integrals includes: the two-loop massive Bhabha scattering in QED, three-loop massless form factors and static potentials, massive two-loop QCD form factors, B-physics studies, hadronic top-quark physics, muon magnetic moment anomaly from the lepton vacuum polarization, the problem of determining the fragmentation functions and parton distributions in QCD analysis of the deep inelastic scattering data and so on.

The problem of finding effective contours for numerical evaluation of MB integrals was formulated in the work [2]. The best efficiency in a numerical integration of the MB integrals can be achieved on the contour of the stationary phase. However, the solution of the differential equation for the stationary phase contour and its subsequent application to calculate the MB integral with high accuracy requires a big computing expenses [3]. Instead, it is proposed to build such approximations of the stationary phase contour that would allow the effective application of the quadrature integration formulas [2, 4]. This requires a detailed study of the system of contours of the stationary phase for various MB integrals. It should be noted that the first attempt to construct an effective approximation for the contour using the expansion of the integrand at the saddle point was made by Kosower [5] as applied to the calculation of parton distributions in the x-space.

The purpose of the project research: *new applications of efficient approximating for the stationary phase contour in calculating of the MB integrals arising in massive Feynman diagrams (see, for example, [2-4]).*

Description of the work for students: *study of the specifics of the numerical estimates of Mellin-Barnes integrals based on the stationary phase contour.*

*Results of the project will be presented in the form of the report and could be consider as a basis of the scientific publication.*

**Acceptance criteria**

The student assumes a basic knowledge of quantum field theory, programming skills MAPLE, MATHEMATICA, LaTeX, and Origin packages.

### **Recommended literature**

1. *Bateman H., Erdelyi A.* Higher transcendental functions. New York: McGraw-Hill, 1953. Vol. 1.
2. *Gluzha J., Jelinski T., Kosower D.A.* Efficient evaluation of massive Mellin-Barnes integrals // *Phys. Rev. D.* Vol. **95**. (2017) Article ID: 076016.
3. *Dubovyk I., Gluzha J., Jelinski T., Riemann T., Usovitsch J.* New prospects for the numerical calculation of Mellin-Barnes integrals in Minkowskian kinematics // *Acta Phys. Polon. B.* Vol. **48** (2017) P. 995–1009.
4. *Sidorov A.V., Lashkevich V.I., Solovtsova O.P.* Asymptotics of the contour of the stationary phase and efficient evaluation of the Mellin-Barnes integral for the  $F_3$  structure function // *Phys. Rev. D.* Vol. **97** (2018) Article ID: 076009.
5. *Kosower D.A., Evolution of parton distributions* // *Nucl. Phys. B.* Vol. **506** (1997) P. 439–467.

### **The expected number of participants of the project**

The number of the participants is 1-2 students in September.

### **The project coordinators from the JINR**

*Bogoliubov Laboratory of Theoretical Physics,*

*<http://www.info.jinr.ru/plan/ptp-2019/a931135.htm>*

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